

LESSON 2.3 – PROBLEM SOLVING





OVERVIEW

Here's what you'll learn in this lesson:

Number and Age

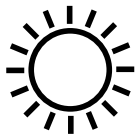
- a. *Translating words into algebraic expressions*
- b. *Number problems*
- c. *Age problems*

Geometry

- a. *Geometry problems*

You may not realize it, but you use algebra every day—whether it's figuring out the least expensive brands to buy in the grocery store or finding the measurements of a fence that will use up some scrap lumber. In fact, you may discover that the more mathematics you know, the more you'll notice it around you.

In this lesson, you will use what you have learned about equations to solve word problems taken from everyday life.



NUMBER AND AGE

Summary

Setting Up Word Problems

One of the most useful aspects of algebra is that it can help you solve problems from everyday life. These problems are often called word problems. Some of the word problems that you will solve deal with numbers, ages, and geometry.

Since many of these word problems have the same basic structure, there are steps you can follow to help you translate the words into an equation which you can then solve.

1. Draw a sketch (when you can).
2. List the quantities to be found. Use English phrases.
3. Represent these quantities algebraically.
4. Write an equation which describes the problem.
5. Solve the equation.
6. Check that the numbers work in the original problem.

Algebra is similar to learning a new language with its own vocabulary and grammar. As you get more familiar with the language of algebra, translating problems from words into equations will become easier.

An Effective Guessing Strategy

Sometimes, guessing can be a good way to help you get started on a problem, especially when you don't know where to begin. If you track your guesses and keep them organized, you can get a sense of the right answer as well as ideas about how to write an equation.

Here is an example. Use guessing as a strategy to help you solve the problem.

The sum of three consecutive integers is 81. Find the three numbers.

Pick any number for your first guess, say your age or the number 10. Then try this number in the problem. If your first guess isn't right, use the information you get when you check your answer to help you make your next guess. Keep refining your guesses.

Checking your answer in the original equation may seem like too much extra work, but it is important to do it to see if your answer makes sense.

first number	next consecutive number	third consecutive number	sum	check?
10	11	12	$10 + 11 + 12 = 33$	too low
20	21	22	$20 + 21 + 22 = 63$	too low
30	31	32	$30 + 31 + 32 = 93$	too high
25	26	27	$25 + 26 + 27 = 78$	too low
26	27	28	$26 + 27 + 28 = 81$	right!

Even if you hadn't been able to guess the right answer, you still might have been able to use the information to write an equation. You could then solve this equation to find the answer. Here's how. Each time you guessed something for the first number, you added 1 to get the second number, and you added 2 to get the third number.

What if x was your guess? What would be the next consecutive number? The third consecutive number?

first number	next consecutive number	third consecutive number	sum	check?
x	$x + 1$	$x + 2$		

If you guess that the first number is x , then the second number is 1 more than x , or $x + 1$; and the third number is 2 more than x , or $x + 2$.

What is the sum of the three numbers?

first number	next consecutive number	third consecutive number	sum	check?
x	$x + 1$	$x + 2$	$x + (x + 1) + (x + 2)$	

How do you want the sum to relate to 81?

first number	next consecutive number	third consecutive number	sum	check?
x	$x + 1$	$x + 2$	$x + (x + 1) + (x + 2) = 81$	

So the equation you need to solve is $x + (x + 1) + (x + 2) = 81$.

When you had three numbers, like 10, 11, and 12, you found their sum by adding them together and then checking to see if the sum was 81. The same method works with the “numbers” x , $(x + 1)$, and $(x + 2)$. In this case, set their sum equal to 81, then solve the equation to find the value of x which makes the equation true.

$$\begin{aligned}x + (x + 1) + (x + 2) &= 81 \\3x + 3 &= 81 \\3x &= 78 \\x &= 26\end{aligned}$$

So the first number is 26, the second number is $(26 + 1) = 27$, and the third number is $(26 + 2) = 28$.

You can check that $26 + 27 + 28 = 81$.

Number Problems

Example 1 Suppose you have two numbers and the second number is 5 more than twice the first. If the sum of the two numbers is 17, what are the numbers?

Let the first number = x .

Then the second number = $5 + 2x$.

The sum of the two numbers is 17.

$$\begin{aligned}x + (5 + 2x) &= 17 \\3x + 5 &= 17 \\3x &= 12 \\x &= 4\end{aligned}$$

So the first number is 4 and the second number is $5 + 2 \cdot 4 = 13$.

You can check that $4 + 13 = 17$.

Example 2 The sum of three consecutive integers is 7 more than twice the largest number. What is the smallest number?

Let the smallest number = x .

Then the next number = $x + 1$.

The third (and largest) number = $x + 2$.

The sum of the three numbers is 7 more than twice the largest number.

$$\begin{aligned}x + (x + 1) + (x + 2) &= 7 + 2(x + 2) \\3x + 3 &= 7 + 2x + 4 \\3x + 3 &= 2x + 11 \\x + 3 &= 11 \\x &= 8\end{aligned}$$

So the smallest number is 8.

You can check that $8 + 9 + 10 = 7 + 2 \cdot 10$.

Notice that this is the same answer as you got in the Guess and Check table.

Age Problems

Example 1 Lloyd is 7 years older than Frank. In 5 years the sum of their ages will be 57. How old are each of them now?

Let Frank's age now = x .

Then Lloyd's age now = $x + 7$.

Frank's age in 5 years = $x + 5$.

Lloyd's age in 5 years = $(x + 7) + 5 = x + 12$.

In 5 years the sum of their ages will be 57.

$$(x + 5) + (x + 12) = 57$$

$$2x + 17 = 57$$

$$2x = 40$$

$$x = 20$$

So Frank is now 20 years old and Lloyd is now $20 + 7 = 27$ years old.

In 5 years, Frank will be 25 and Lloyd will be 32. You can check that $25 + 32 = 57$.

Example 2 Tara is one-fifth her father's age now. In 8 years Tara will be the same age as her father was 16 years ago. How old is each of them now?

Let Tara's age now = x .

Then her father's age now = $5x$.

Tara's age in 8 years = $x + 8$.

Her father's age 16 years ago = $5x - 16$.

In 8 years Tara will be the same age as her father was 16 years ago.

$$x + 8 = 5x - 16$$

$$x + 24 = 5x$$

$$24 = 4x$$

$$6 = x$$

So Tara is now 6 years old and her father is $5 \cdot 6 = 30$ years old.

In 8 years Tara will be 14. 16 years ago her father was 14.

Sample Problems

1. One number is 11 less than 7 times another. If their sum is 21, what are the two numbers?
- a. List the quantities you want to find. two numbers
- b. Represent these quantities algebraically. the first number = x
the second number = _____
- c. Write an equation to describe the problem. _____ + _____ = 21
- d. Solve the equation. _____ - 11 = ____
_____ x = ____
 x = ____
So the first number = ____ and the second number = ____.
- e. Check that the numbers work in the original problem. Is ____ + ____ = 21?
Is ____ = 21? ____

2. Rick is three times as old as Holly. Three years ago, he was four times as old as Holly. How old is each of them now?

- a. List the quantities you want to find. Holly's age
Rick's age
- b. Represent these quantities algebraically. Holly's age = x
Rick's age = ____
Holly's age 3 years ago = $x - 3$
Rick's age 3 years ago = _____
- c. Write an equation which describes the problem. $4(x - 3) =$ _____
- d. Solve the equation. $4x -$ ____ = ____
 $4x -$ ____ + ____ = ____ + ____
 $4x = 3x + 9$
 $4x - 3x = 3x + 9 -$ ____
 $x =$ ____
- e. Check that the numbers work in the original problem. So Holly is now _____ years old and Rick is _____ years old, so he is three times as old as she. Three years ago, Holly was ____ and Rick was ____, so he was then 4 times her age.

Answers to Sample Problems

b. $7x - 11$

c. $x, 7x - 11$ (in either order)

d. $8x, 21$

8, 32

4

4

17

e. 4, 17

21, Yes

b. $3x$

$3x - 3$

c. $3x - 3$

d. 12, $3x - 3$

12, 12, $3x - 3$

$3x$

9

e. 9

27

6

24

GEOMETRY

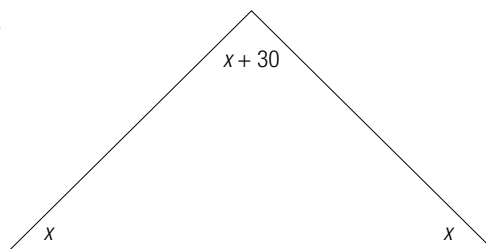
Summary

Solving Geometry Problems

To solve a geometry problem you can use the same basic strategies you used to solve number and age problems. In addition, you may need to use a geometric relationship which is not explicitly stated. If a geometry problem does not seem to have enough information, think about whether there is some geometric relationship which may help.

Example 1 In an isosceles triangle (which has two equal angles), the measure of one angle is 30 degrees larger than the measure of each of the other two. What is the measure of each angle?

Draw a sketch.



Let the measure of each of the two equal angles = x .

Then the measure of the other angle = $x + 30$.

To solve this problem, you need to use a geometric relationship that was not given.

the sum of the measures of the angles in a triangle = 180 degrees

$$x + x + (x + 30) = 180$$

$$3x + 30 = 180$$

$$3x = 150$$

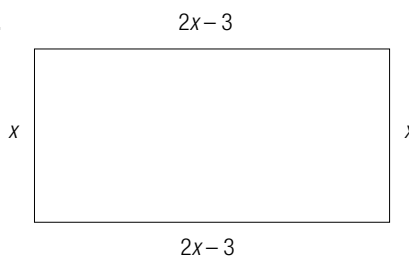
$$x = 50$$

So the measure of each of the two equal angles = 50 degrees, and the measure of the third angle = $50 + 30 = 80$ degrees.

You can check that $50 + 50 + 80 = 180$.

Example 2 The length of a rectangle is 3 feet less than twice its width. If the perimeter of the rectangle is 30 feet, find its length and width.

Draw a sketch.



Let the width = x .

Then the length = $2x - 3$.

To solve this problem, you need to use a geometric relationship that was not given.

the perimeter of a rectangle = width + length + width + length

The perimeter of the rectangle is 30 feet.

$$x + (2x - 3) + x + (2x - 3) = 30$$

$$6x - 6 = 30$$

$$6x = 36$$

$$x = 6$$

So the width of the rectangle is 6 feet and the length is $2 \cdot 6 - 3 = 9$ feet.

You can check that $6 + 9 + 6 + 9 = 30$.

Sample Problems

1. The length of a rectangular yard is 12 feet more than twice its width. If the fence which encloses the yard is 276 feet long, what are the dimensions of the yard?

- a. Draw a picture.



- b. List the quantities you want to find.

width
length

- c. Represent these quantities algebraically.

width = x
length = _____

- d. Write an equation which describes the problem.

$$\text{____} + \text{____} + \text{____} + \text{____} = 276$$

- e. Solve the equation.

$$6x + \text{____} = \text{____}$$

$$6x = \text{____}$$

$$x = \text{____}$$

So the width of the yard is _____ feet and the length of the yard is _____ feet.

- f. Check that the numbers work in the original problem.

If the width of the yard is _____ feet then the length of the yard is _____ feet, which is 12 feet longer than twice the width. If the width is _____ feet and the length is _____ feet, then the perimeter of the yard is _____ feet.

Answers to Sample Problems

c. $2x + 12$

d. $x, 2x + 12, x, 2x + 12$ (in any order)

e. 24, 276

252

42

42

96

f. 42

96

42

96

276



HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



Explain

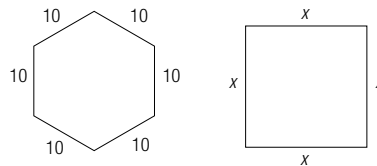
Number and Age

1. The sum of four consecutive integers is -118 . What are the four numbers?
2. The sum of two consecutive integers is -1 . What is the larger interger?
3. The sum of three consecutive odd integers is 81 . What are the three integers?
4. Latoya is twice as old as her cousin was 3 years ago. If the sum of their ages now is 15 , how old is each one of them?
5. Mount Everest is the tallest mountain in the world. It is 237 meters higher than K2, the second tallest mountain. If the sum of their heights is $17,459$ meters, how tall is each mountain?
6. Eleven years ago Hye was four times as old as her brother. In 1 year she will be twice as old as he is now. What are their ages now?
7. A molecule of propane has 26 atoms. If there are 6 fewer hydrogen atoms than 3 times the number of carbon atoms, how many atoms of each does it contain?
8. Ariel is 2 years older than twice Juan's age and Felix is 6 years older than Juan. If the sum of their ages is 80 , how old is each person?
9. One number is 9 more than 3 times another. If their sum is 53 , what is the smaller number?
10. When John F. Kennedy was sworn in as President, he was 1 year older than Teddy Roosevelt was when Roosevelt took the office. If the sum of their ages when each became President was 85 , how old was Kennedy when he was sworn in?

11. The average surface temperature on Earth (in degrees Celsius) is 70 degrees more than the average surface temperature on Mars. If the sum of the average temperatures on the two planets is -20 degrees, what is the average surface temperature on Mars?
12. Toshihiko is 4 years more than twice as old as Kyoko. If the sum of their ages is 79 , how old is Kyoko?

Geometry

13. The length of the longest leg of a triangle is twice the length of the shortest leg. The remaining leg is 2 inches longer than the shortest leg. If the perimeter of the triangle is 26 inches, how long is each leg?
14. A regular hexagon (which has 6 sides all the same length) has the same perimeter as a square. If the length of a side of the hexagon is 10 centimeters, how long is one side of the square?



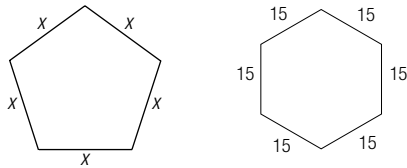
15. The length of a rectangle is 7 inches less than 3 times its width. If the perimeter of the rectangle is 50 inches, what are its dimensions?
16. The distance around one circular track is three times as far as the distance around a second circular track. If the sum of the distances around both tracks is 80π yards, what is the distance around the larger track?

17. If one side of a square is increased by 11 feet and an adjacent side is decreased by 5 feet, a rectangle is formed whose perimeter is 52 feet. Find the length of a side of the original square.

18. The measure of the smallest angle of a right triangle is 15 degrees less than half the measure of the next smallest angle. What is the measure of each angle?

19. The length of a rectangle is 12 feet less than three times its width. If the perimeter of the rectangle is 24 feet, what are its dimensions?

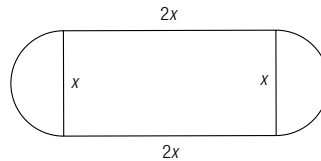
20. A regular pentagon (which has 5 sides all the same length) has the same perimeter as a regular hexagon (which has 6 sides all the same length). If the length of a side of the hexagon is 15 feet, how long is one side of the pentagon?



21. When one side of an isosceles triangle (which has two equal sides) is increased by 3 inches, the triangle becomes an equilateral triangle (which has three equal sides). If the perimeter of the triangle is initially 18 inches, how long is each side of the original triangle?

22. The measure of one angle of a triangle is 20 degrees more than the measure of the smallest angle. The measure of another angle is 8 degrees less than twice the measure of the smallest angle. What is the measure of each angle?

23. A rectangular track was being built so that the length of one of the short sides was half the length of one of the long sides. The track was supposed to be 300 yards around. At the last minute, the plans for the track were changed and a semicircle (half a circle) was added at each of the short ends. What is the distance around the track after the plans were modified?



24. A regular octagon (which has 8 sides all the same length) has the same perimeter as a regular hexagon (which has 6 sides all the same length). If the length of one of the sides of the octagon is 1 inch less than a side of the hexagon, what is the length of a side of each figure?



APPLY

Practice Problems

Here are some additional practice problems for you to try.

Number and Age

1. The sum of two numbers is 42. One number plus 2 times the other number is 57. What are the numbers?
2. The sum of two numbers is 43. One number plus three times the other number is 65. What are the numbers?
3. The sum of two numbers is 45. Their difference is 9. What are the numbers?
4. The sum of two numbers is 24. Their difference is 52. What are the numbers?
5. The sum of two numbers is 16. Their difference is 40. What are the numbers?
6. The difference between two numbers is 55. Four times the smaller number plus five times the larger number is 176. What are the numbers?
7. The difference between two numbers is -38 . Two times the smaller number minus five times the larger number is -217 . What are the numbers?
8. The difference between two numbers is 80. Three times the smaller number plus four times the larger number is -254 . What are the numbers?
9. The sum of three consecutive integers is 96. What are the numbers?
10. The sum of four consecutive integers is -226 . What are the numbers?
11. The sum of four consecutive integers is 114. What are the numbers?
12. The sum of three consecutive even integers is 444. What are the numbers?
13. The sum of four consecutive even integers is -316 . What are the numbers?
14. The sum of four consecutive odd integers is -32 . What are the numbers?
15. David is 3 years older than Sean. The sum of their ages is 15. How old is Sean?
16. Alexandra is 8 years younger than Natasha. The sum of their ages is 30. How old is Alexandra?
17. Jeremy is six years older than Barbara. The sum of their ages is 68. How old is Barbara?
18. Carl is 9 years older than his cousin Jenny. If the sum of their ages is 77, how old is each one of them?
19. Miriam is ten years younger than her husband Edward. If the sum of their ages is 106, how old is each one of them?
20. Pietro is 12 years younger than Annetta. If the sum of their ages is 62, how old is each one of them?
21. Mark is three times as old as Luke. In 5 years Mark will be two times as old as Luke is in 5 years. How old is each one now?
22. Serge is five times as old as his daughter Katia. In 12 years Serge will be three times as old as Katia is in 12 years. How old is each one now?
23. Svetlana is four times as old as Boris. In 10 years Svetlana will be three times as old as Boris is in 10 years. How old is each one now?
24. Brandon is three times as old as Caitlin. Eighteen years ago, Brandon was six times as old as Caitlin was eighteen years ago. How old is each one now?
25. Masato is twice as old as Kim. Ten years ago, Masato was three times as old as Kim was ten years ago. How old is each one now?

26. Gerhard is twice as old as Isolde. Sixteen years ago, Gerhard was four times as old as Isolde was sixteen years ago. How old is each one now?
27. In 7 years, Maria will be four times as old as Angelica will be then. The sum of their ages now is 71. How old will each of them be in 5 years?
28. In 5 years, Alessandro will be three times as old as Frederico will be then. The sum of their ages now is 86. How old will each of them be in 3 years ?

Geometry

29. An isosceles triangle has two angles whose measures are equal. If the largest angle of the triangle measures 85 degrees, what are the measures of the other two angles?
30. If the largest angle of an isosceles triangle measures 68 degrees, what are the measures of the other two equal angles?
31. The sum of the angle measures of a triangle is 180° . The smallest angle in a triangle is 64 degrees less than the measure of the largest angle. The measure of the remaining angle is 8 degrees more than the measure of the smallest angle. What is the measure of each angle?
32. The measure of the smallest angle in a triangle is 50 degrees less than the measure of the largest angle. The measure of the remaining angle is 10 degrees more than the measure of the smallest angle. What is the measure of each angle?
33. The measure of the largest angle in a triangle is 55 degrees more than the smallest angle. The measure of the remaining angle is 5 degrees less than the measure of the largest angle. What is the measure of each angle?
34. The shortest side of a triangle is 3 inches shorter than the longest side. The remaining side is 2 inches longer than the shortest side. The perimeter of the triangle is 20 inches. What is the length of each side? (Note: The perimeter of a figure is the distance around the outside of the figure.)
35. The longest side of a triangle is 12 cm longer than the shortest side. The remaining side is 2 cm shorter than the longest side. The perimeter of the triangle is 31 cm. What is the length of each side?
36. The longest side of a triangle is 7 cm longer than the shortest side. The remaining side is 3 cm shorter than the longest side. The perimeter of the triangle is 29 cm. What is the length of each side?
37. The shortest side of an isosceles triangle is 4 cm shorter than the length of each of the equal sides. The perimeter of the triangle is 26 cm. What is the length of each side?
38. The shortest side of an isosceles triangle is 5 inches shorter than the length of each of the equal sides. The perimeter of the triangle is 43 inches. What is the length of each side?
39. The shortest side of an isosceles triangle is half the length of each of the equal sides. The perimeter of the triangle is 80 inches. What is the length of each side?
40. The length of a rectangle is 10 cm longer than its width. The perimeter of the rectangle is 68 cm. What are the length and width of the rectangle? (Note: The perimeter of a rectangle is the distance around the outside of the rectangle.)
41. The width of a rectangle is 4 inches shorter than its length. The perimeter of the rectangle is 36 inches. What are the length and width of the rectangle?
42. The width of a rectangle is 9 cm shorter than its length. The perimeter of the rectangle is 40 cm. What are the length and width of the rectangle?
43. The length of a rectangle is 10 cm less than five times its width. The perimeter of the rectangle is 52 cm. What are the length and width of the rectangle ?
44. The length of a rectangle is 23 cm less than three times its width. The perimeter of the rectangle is 82 cm. What are the length and width of the rectangle?
45. The length of a rectangle is 2 inches more than twice its width. The perimeter of the rectangle is 28 inches. What are the length and width of the rectangle?
46. The width of a rectangle is 52 inches less than four times its length. The perimeter of the rectangle is 51 inches. What are the length and width of the rectangle?
47. The width of a rectangle is 25 inches less than 3 times its length. The perimeter of the rectangle is 38 inches. What are the length and width of the rectangle?

48. The width of a rectangle is 3 more than half its length. The perimeter of the rectangle is 60 cm. What are the length and width of the rectangle?
49. The perimeter of an equilateral triangle (which has three sides, all the same length) is four times the perimeter of a regular hexagon (which has 6 sides, all the same length). The length of a side of the triangle is 10 cm more than six times the length of a side of the hexagon. What is the perimeter of the triangle? What is the perimeter of the hexagon?
50. The perimeter of a square is three times the perimeter of a regular hexagon (which has 6 sides all the same length). The length of a side of the square is 2 inches more than four times the length of a side of the hexagon. What is the perimeter of the square? What is the perimeter of the hexagon?
51. The length of a rectangular playground is four times its width. The perimeter of the playground is 250 feet. What is the area of the play ground? (Note: The area of a rectangle is found by multiplying its length by its width.)
52. The length of a rectangular park is five times its width. The perimeter of the park is 108 miles. What is the area of the park?
53. The length of a rectangular floor is six times its width. The perimeter of the floor is 210 feet. What is the area of the floor?
54. The length of a rectangular pool is 4 m more than twice its width. The perimeter of the pool is 20 m. What is the area of the pool?
55. The length of a rectangular garden is 3 feet more than twice its width. The perimeter of the garden is 78 feet. What is the area of the garden?
56. The width of a rectangular window is 10 feet less than twice its length. The perimeter of the window is 28 feet. What is the area of the window?



Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. One number is 3 more than another. Twice the larger number minus the smaller number is 15. What are the two numbers?
2. Abe and his younger sister are 3 years apart in age. If the sum of their ages will be 35 next year, what are their ages now?
3. Five years ago, Felipe was half of Carolina's age. At that time, the sum of their ages was 30. How old is Felipe now?
4. The sum of three consecutive odd integers is 5 less than 4 times the smallest such integer. What are the three odd integers?
5. A rectangular park was built so that its length is 3 times its width. The perimeter of the park is 24 yards. What are the width and length of the park?
6. The measure of one angle of a triangle is 10 degrees more than the measure of the smallest angle. The measure of the third angle is 50 degrees more than the measure of the smallest angle. What are the measures of the angles of the triangle?
7. The distance around a rectangular city block is 280 yards. If the length of the block is 10 yards less than twice its width, what are the dimensions of the block?
8. The perimeter of a certain square is the same as the perimeter of a certain equilateral triangle. (An equilateral triangle is a triangle in which all three sides have the same length.) Each side of the triangle is 1 inch longer than a side of the square. How long is a side of the square? How long is a side of the triangle?

