$A = \pi r^2$

$V = \pi r^2 h$
Overview

You have learned how to identify lines, line segments, and rays and to name different types of polygons. You have also learned how to measure and classify angles.

In this lesson, you will learn how to find the perimeter and the area of certain polygons. Then, you will study 3–D objects including spheres, rectangular prisms, cylinders and cones. Finally, you will learn how to find the surface area and volume of some of these 3–D objects.

Before you begin, you may find it helpful to review the following mathematical ideas which will be used in this lesson:

**Review 1**

Multiply fractions.

Do this multiplication: \( \frac{22}{7} \times 6 \)

*Answer: \( \frac{132}{7} \)*

**Review 2**

Multiply decimal numbers.

Do this multiplication: 3.14 x 6

*Answer: 18.84*

**Review 3**

Use the order of operations.

Do this calculation: \( \frac{1}{2} \times (7.8 + 9.6) + 5 \)

*Answer: 13.7*
In Concept 1: Area and Perimeter, you will find a section on each of the following:

- Perimeter of a Polygon
- Area of a Rectangle
- Area of a Parallelogram
- Area of a Triangle
- Area of a Trapezoid
- Circumference and Area of a Circle

The word **perimeter** comes from two Greek words:

\[
\text{PERI} + \text{METER}
\]

“around”  “to measure”

So perimeter means the “measure around” a figure.

A ruler is often used to measure the lengths of the sides of a polygon.

---

### Example 1

1. Find the perimeter of the quadrilateral in Figure 1.

![Figure 1](image1.png)

To find the perimeter of this polygon:

- Add the lengths of its sides.

\[
4.5 \text{ cm} + 5.5 \text{ cm} + 5.9 \text{ cm} + 3.2 \text{ cm} = 19.1 \text{ cm}
\]

So, the perimeter of the quadrilateral is 19.1 cm.

---

### Example 2

2. Find the perimeter of the triangle in Figure 2.

![Figure 2](image2.png)
To find the perimeter of this triangle:

- Add the lengths of its sides. 
  \[ \frac{5}{8} \text{ in.} + \frac{1}{4} \text{ in.} + \frac{7}{8} \text{ in.} = \frac{3}{4} \text{ in.} \]

So, the perimeter of the triangle is \( \frac{3}{4} \) in.

A way to add the mixed numbers:

\[
\begin{align*}
\frac{5}{8} + \frac{1}{4} + \frac{7}{8} &= \frac{5}{8} + \frac{2}{8} + \frac{7}{8} \\
&= \frac{5 + 2 + 7}{8} \\
&= \frac{14}{8} = \frac{5}{4} = \frac{3}{4}
\end{align*}
\]

3. The perimeter of the pentagon in Figure 3 is 18.6 cm. Find the missing length.

To find the missing length:

- Add the given lengths. 
  \[ 2.7 + 2.7 + 5.2 + 3.1 = 13.7 \]

- Subtract that result from the perimeter. 
  \[ 18.6 - 13.7 = 4.9 \]

So, the missing length is 4.9 cm.

**Example 3**

**Area of a Rectangle**

Another measurement associated with polygons is area.

The area of a polygon is the size of the region “inside” the polygon. One way to measure the area of a polygon is to choose a shape and find out how many of those shapes exactly cover the “inside” of the polygon.

For example, it takes approximately 15 circles of the given size to cover the inside of the polygon shown in Figure 4.

![Figure 4](image)

So, the area of the polygon is approximately 15 of those circles.

A square of a certain size is the shape that is used most often to cover a polygon.

For example, in Figure 5, the inside of the polygon is covered exactly by 15 squares. (Some of the squares are broken in half.) Here, each square has size 1 cm by 1 cm. That’s a square centimeter.

![Figure 5](image)

Caution! Perimeter is measured in ruler units such as inches, feet, centimeters, meters, and so on.

Area is measured in **square units** such as square inches, square feet, square miles, square centimeters, square meters, and so on.
Notice that when the area of a polygon is written, the number of squares and the size of each square is included.

Now you will see how to find the area of a special polygon, a rectangle.

A rectangle is a 4–sided polygon with 4 right angles. Two sides that form a right angle are called perpendicular sides.

Figure 6

One side of a rectangle is called the base. The base is usually the side the rectangle “rests” on. See Figure 6. “Base” can also mean the length of that side.

The height is the perpendicular distance between the base and the opposite side. See Figure 6.

A square is a special type of rectangle. In a square, all 4 sides have the same length.

To find the area of a rectangle, multiply: base times height.

\[ \text{Area of a rectangle} = \text{base} \times \text{height} \]

\[ A = bh \]

4. Find the area of the rectangle in Figure 7.

Figure 7

To find the area of the rectangle:

• Multiply the base by the height.

\[ A = bh \]

\[ = 7 \text{ cm} \times 2\text{ cm} \]

\[ = 14 \text{ cm}^2 \]

So, the area of the rectangle is 14 cm$^2$.
5. In Figure 8 each side of the square is 3.6 cm. Find the area of the square.

To find the area of the square:

- Multiply the base and the height. Area = base \times height
- The base and height are each equal to the length of a side. So the area of the square is:
- Do the multiplication. 

So, the area of the square is 12.96 cm².

6. In Figure 9 the base of the rectangle measures 9 units. The area of the rectangle is 54 square units. Find the height of the rectangle.

Here’s a way to find the height of the rectangle:

- Use the formula: Area = base \times height
  - Replace “Area” with 54.
  - Replace “base” with 9.
  - Since 54 = 9 \times height, height = 6

So, the height of the rectangle is 6 units.

**Parallellograms**

You have seen how to find the area of a rectangle. Now you will see how to find the area of a parallelogram.

A parallelogram is a 4-sided polygon in which the opposite sides are parallel. See Figure 10.

In a parallelogram notice that opposite sides have the same length.
Every rectangle is a parallelogram. But, **not** every parallelogram is a rectangle.

Caution! To find the height of a parallelogram, do not measure a side. Instead, measure along a line segment that makes a right angle with the base.

For a parallelogram, you can picture any side as the base. But once you choose the base, you must measure the height between that base and the opposite side.

To find the area of a parallelogram, you multiply the base by the height, just like you do for a rectangle. But always make sure that the height, \( h \), is the “right angle distance” between the base and the opposite side.

To find the area of a parallelogram:

- Multiply the base by the height.

\[
A = bh
\]

Notice that the measurement of \( 2 \frac{1}{4} \text{ in.} \) is not needed to calculate the area of the given parallelogram.

7. Find the area of the parallelogram in Figure 13.

To find the area of the parallelogram:

- Multiply the base and the height.

\[
A = bh = 4 \frac{1}{4} \text{ in.} \times 1 \frac{7}{8} \text{ in.} = 7 \frac{31}{32} \text{ in}^2
\]

So, the area of the parallelogram is \( 7 \frac{31}{32} \text{ in}^2 \).
8. In Figure 14 find the area of the parallelogram.

To find the area of the parallelogram:

- Multiply the base and the height.

\[ A = bh \]

\[ = 3.4 \times 5.1 \text{ cm} \]

\[ = 17.34 \text{ cm}^2 \]

So, the area of the parallelogram is 17.34 cm².

9. The area of the parallelogram in Figure 15 is 24 square units. Here, the base of the parallelogram is 4 units. Find the height of the parallelogram.

Here’s a way to find the height of the parallelogram.

- Use the formula: \( \text{Area} = \text{base} \times \text{height} \)

Replace “Area” with 24.
Replace “base” with 4.
Since \( 24 = 4 \times \text{height} \), \( \text{height} = 6 \)

So, the height of the parallelogram is 6 units.

**Triangles**

Now, you will see how to find the area of a triangle.

Recall that a triangle is a 3-sided polygon.

A triangle, like a rectangle or a parallelogram, has a base and a height (or altitude).

As shown in Figure 16, the base can be any side of the triangle. “Base” can also mean the length of that side.
The height of the triangle is the perpendicular distance between the base and the opposite “corner.” As shown in Figure 17, sometimes you have to extend the base to measure the height.

In a triangle, you can picture any side as the base. But once you choose the base, you have to measure the height between that base and the opposite angle.

**Figure 17**

It is easy to construct a parallelogram out of a triangle. To do this, you copy the triangle and arrange the two triangles to form a parallelogram. See Figure 18.

**Figure 18**

Notice that the parallelogram is made up of two triangles of the same size. So the area of each triangle is half the area of the parallelogram.

To find the area of a triangle, multiply: one–half times the base times the height.

\[
\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}
\]

\[
\text{Area} = \frac{1}{2} \times b \times h
\]

\[
A = \frac{1}{2}bh
\]

---

**Example 10**

10. The triangle in Figure 19 has base 8.2 cm and height 3.1 cm.

Find the area of the triangle.

**Figure 19**

To find the area of the triangle:

- Multiply \(\frac{1}{2}\) times the base times the height.

\[
A = \frac{1}{2} \times 8.2 \, \text{cm} \times 3.1 \, \text{cm}
\]

\[
A = 12.71 \, \text{cm}^2
\]

---

**Example 11**

11. The triangle in Figure 20 has base \(2\frac{1}{8}\) in. and height \(1\frac{1}{4}\) in.

Find the area of the triangle.

**Figure 20**
To find the area of the triangle:

- Multiply \( \frac{1}{2} \) times the base times the height.

\[
A = \frac{1}{2}bh
\]

\[
= \frac{1}{2} \times \frac{11}{8} \text{ in} \times \frac{1}{4} \text{ in}
\]

\[
= \frac{1}{2} \times \frac{17}{8} \text{ in} \times \frac{5}{4} \text{ in.}
\]

\[
= \frac{85}{64} \text{ in}^2
\]

That's \( \frac{21}{64} \) \text{ in}^2.

So, the area of the triangle is \( \frac{21}{64} \text{ in}^2 \).

12. In Figure 21, the area of the triangle is 18 square units. Here, the base of the triangle is 9 units. Find the height of the triangle.

Here’s a way to find the height:

- Use the formula: \[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]

Replace "Area" with 18.

Replace "base" with 9.

Multiply both sides by 2.

Since 36 = 9 \times 4, \text{ height} = 4

So, the height of the triangle is 4 units.

**Trapezoids**

Now you will see how to find the area of a trapezoid.

A trapezoid is a 4-sided polygon that has two parallel sides.

In a trapezoid, the two parallel sides are called bases.

As shown in Figure 22, the height of the trapezoid is the perpendicular distance between the bases.
We can use what we know about the area of a parallelogram to find the area of a trapezoid. First, we transform a trapezoid into a parallelogram. To do this, you copy the trapezoid and arrange the two trapezoids to form a parallelogram. See Figure 23.

Notice that the parallelogram is made up of two trapezoids of the same size. So the area of the trapezoid is one half the area of the parallelogram.

$$\text{Area of the trapezoid} = \frac{1}{2} \times \text{Area of the parallelogram}$$
$$= \frac{1}{2} \times \text{base of parallelogram} \times \text{height of parallelogram}$$
$$= \frac{1}{2} \times (\text{base 1 of trapezoid} + \text{base 2 of trapezoid}) \times \text{height}$$

To find the area of a trapezoid:
- Add the bases.
- Multiply: one-half times the sum of the bases times the height.

$$\text{Area of a trapezoid} = \frac{1}{2} \times (\text{base}_1 + \text{base}_2) \times \text{height}$$

$$A = \frac{1}{2} (b_1 + b_2) h$$

13. Find the area of the trapezoid in Figure 24.

Here’s one way to find the area of the trapezoid:
- Add the bases. 11 units + 9 units = 20 units
- Multiply that result by the height. 20 units \(\times\) 5 units = 100 square units
- Multiply by \(\frac{1}{2}\). 

So, the area of the trapezoid is 50 square units.
14. Find the area of the trapezoid in Figure 24.

Here’s another way to find the area of the trapezoid:

- Add the bases and divide by 2. \( \frac{9 \text{ units} + 11 \text{ units}}{2} = 10 \text{ units} \)
  (That’s the average length of the bases.)

- Multiply that result by the height. \( 10 \text{ units} \times 5 \text{ units} = 50 \text{ square units} \)

So, the area of the trapezoid is 50 square units.

15. In Figure 25, the trapezoid has area 21 cm\(^2\).

Find the height of the trapezoid.

Here’s one way to find the height of the trapezoid:

- Use the area formula. \( A = \frac{1}{2} \times (b_1 + b_2) \times h \)
  Replace \( A \) by 21 cm\(^2\). \( 21 \text{ cm}\(^2\) = \frac{1}{2} \times (5 \text{ cm} + 9 \text{ cm}) \times h \)
  Replace \( b_1 \) by 5 cm and \( b_2 \) by 9 cm. \( 21 \text{ cm}\(^2\) = \frac{1}{2} \times 14 \text{ cm} \times h \)
  Divide both sides by 7 cm. \( 3 \text{ cm} = h \)

So, the height of the trapezoid is 3 cm.

---

**Circles**

Now you will work with circles.

A circle is not a polygon because a circle is not made up of straight line segments.

A circle may be named using its center. The circle in Figure 26 is called Circle A since its center is the point A.

The radius of a circle is the distance from its center to any point on the circle. As shown in Figure 27, “radius” also means any line segment joining the center to a point on the circle.

The diameter of a circle is the distance across the circle through the center of the circle. As shown in Figure 28, “diameter” also means any line segment that goes through the center with both endpoints on the circle.
Any diameter of a circle consists of 2 radii that form a straight line segment.

So, a diameter is 2 times as long as a radius. This is written:

\[ d = 2 \times r \]
\[ d = 2r \]

Given the radius, you can use this formula to find the diameter or given the diameter, you can find the radius of a circle.

For example, if the radius of a certain circle is 6 inches, what is the diameter of the circle?

To find the diameter:

• Use the formula \( d = 2r \)
  
Replace \( r \) by 6 inches.

\[ d = 2 \times 6 \text{ inches} \]
\[ = 12 \text{ inches} \]

So, the diameter of the circle is 12 inches.

**Circumference of a Circle**

Now you will see how to find the distance around a circle, the perimeter.

The "perimeter" is called the circumference of the circle.

To find the circumference of a circle, you multiply its diameter by a special number that’s a bit more than 3. This number is called \( \pi \) (that’s the Greek letter “pi”).

To find the circumference of a circle, multiply its diameter by \( \pi \).

The circumference of a circle is \( \pi \) times the diameter:

\[ C = \pi \times d \]
\[ C = \pi d \]

Since the diameter of a circle is twice its radius, the circumference of a circle is also given by:

\[ C = \pi \times (2r) \]
\[ C = 2\pi r \]

You can use the decimal number 3.14 or the fraction \( \frac{22}{7} \) to estimate \( \pi \).

These estimates are used to find the approximate circumference of a circle.

---

16. In Figure 29, the diameter of the circle is 4 centimeters. Find the circumference of the circle.

![Figure 29](image)

To find the circumference of the circle:

• Multiply the diameter by \( \pi \).

\[ C = \pi d \]
\[ = \pi \times 4 \text{ cm} \]
\[ = 4\pi \text{ cm} \]

So, the circumference of the circle is \( 4\pi \) cm.
17. In Figure 30, a circle with radius 2.4 centimeters is shown. Find the circumference of the circle. (Use 3.14 to approximate pi.)

Here’s a way to find the circumference of the circle:

- Multiply the radius by $2\pi$.
  
  $C = 2\pi r$
  
  $= 2 \times \pi \times r$

Use 3.14 to approximate $\pi$.

$\approx 2 \times 3.14 \times 2.4 \text{ cm}$

$= 15.072 \text{ cm}$

So, the circumference of the circle is approximately 15.072 cm.

18. In Figure 31, a circle with diameter $2\tfrac{1}{4}$ inches is shown. Find the circumference of the circle. (Use $\tfrac{22}{7}$ to approximate pi.)

Here’s a way to find the circumference of the circle:

- Multiply the diameter by $\pi$.
  
  $C = \pi d$
  
  $= \pi \times d$

Use $\tfrac{22}{7}$ to approximate $\pi$.

$\approx \tfrac{22}{7} \times 2\tfrac{1}{4} \text{ in}$

$= \tfrac{22}{7} \times \tfrac{9}{4} \text{ in}$

$= \tfrac{99}{14} \text{ in}$

$= 7\tfrac{1}{14} \text{ in}$

So, the circumference of the circle is approximately $7\tfrac{1}{14}$ inches.

Area of a Circle

You have just seen how to find the circumference, the distance around a circle. You can do this using its radius and the number $\pi$, or its diameter and the number $\pi$.

To find the area of a circle:

- Multiply the radius by itself.
- Multiply that result by $\pi$.

The area of a circle is $\pi$ times the radius squared.

$\text{Area} = \pi \times r \times r$

$A = \pi r^2$

$A = \pi r^2$
Example 19

19. In Figure 32, a circle with radius \( \frac{3}{4} \) inch is shown. Find the area of the circle. (Use \( \frac{22}{7} \) to approximate \( \pi \).)

To find the area of the circle:

- Multiply the radius by itself.
  \[ \frac{3}{4} \text{ in} \times \frac{3}{4} \text{ in} = \frac{9}{16} \text{ in}^2 \]
- Multiply that result by \( \pi \).
  \[ A = \pi \times \frac{9}{16} \text{ in}^2 \]
  \[ = \frac{22}{7} \times \frac{9}{16} \text{ in}^2 \]
  \[ = \frac{99}{56} \text{ in}^2 \]
  \[ = \frac{43}{56} \text{ in}^2 \]

So, the area of the circle is approximately \( \frac{43}{56} \text{ in}^2 \).

Example 20

20. In Figure 33, the circumference of the circle shown is \( 6\pi \) cm. Find the area of the circle.

Here’s one way to find the area of the circle:

- Find the radius of the circle.
  - Use the circumference formula.
    \[ C = 2\pi r \]
    Replace \( C \) by \( 6\pi \).
    \[ 6\pi \text{ cm} = 2\pi r \]
    Divide both sides by \( 2\pi \).
    \[ 3 \text{ cm} = r \]
  So, the radius of the circle is 3 cm.
- Use the area formula.
  \[ A = \pi r^2 \]
  Replace \( r \) by 3.
  \[ = \pi \times 3 \text{ cm} \times 3 \text{ cm} \]
  \[ = 9\pi \text{ cm}^2 \]

So the area of the circle is \( 9\pi \text{ cm}^2 \).

Example 21

21. Find the area of the object shown in Figure 34, (Use 3.14 to approximate \( \pi \).)

You may find these Examples useful while doing the homework for this section.
Here’s one way to find the area of the object:

• Deconstruct the picture as shown in Figure 35.

![Figure 35](image)

• The area of the object is the area of the triangle plus the area of the semicircle.

  — Find the area of the triangle. \[ A = \frac{1}{2} bh \]
  \[ = 0.5 \times 2.4 \text{ cm} \times 2.6 \text{ cm} \]
  \[ = 3.12 \text{ cm}^2 \]

  — Find the area of the semicircle. \[ A = \frac{1}{2} \pi r^2 \]
  (Use 3.14 to approximate \(\pi\).)
  \[ = 0.5 \times 3.14 \times 1.2 \text{ cm} \times 1.2 \text{ cm} \]
  \[ = 2.2608 \text{ cm}^2 \]

  — Add the two areas. Area of the object \[ \approx 3.12 \text{ cm}^2 + 2.2608 \text{ cm}^2 \]
  \[ = 5.3808 \text{ cm}^2 \]

So, the area of the object is approximately 5.3808 cm².

22. In Figure 36 find the area and the perimeter of the shaded region shown.

(Use 3.14 to approximate \(\pi\).)

![Figure 36](image)

Here’s one way to find the area and the perimeter of the shaded region:

• Deconstruct the picture as shown in Figure 37.

![Figure 37](image)

• The area of the object is the area of the parallelogram minus the area of the semicircle.

  — Find the area of the parallelogram. \[ A = bh \]
  \[ = 6 \text{ in} \times 3 \text{ in} \]
  \[ = 18 \text{ in}^2 \]

  Find the area of the semicircle. \[ A = 0.5 \pi r^2 \]
  (Use 3.14 to approximate \(\pi\).)
  \[ \approx 0.5 \times 3.14 \times 2 \text{ in} \times 2 \text{ in} \]
  \[ = 6.28 \text{ in}^2 \]

  Subtract the area of the semicircle from the area of the parallelogram.
  Area of the object \[ \approx 18 \text{ in}^2 - 6.28 \text{ in}^2 \]
  \[ = 11.72 \text{ in}^2 \]

So, the area of the shaded region is approximately 11.72 in².
The perimeter of the object is the sum of the straight edges of the parallelogram plus the perimeter of the semicircle.

- Find the sum of the straight edges of the parallelogram.
  \[ P = 5 \text{ in} + 6 \text{ in} + 5 \text{ in} + 1 \text{ in} + 1 \text{ in} = 18 \text{ in} \]

- Find the perimeter of the semicircle.
  \[ P = \frac{1}{2} \pi d \]

  (Use 3.14 to approximate \( \pi \).)

  \[ 0.5 \times 3.14 \times 4 \text{ in} = 6.28 \text{ in} \]

  Add the two distances.

  \[ \text{Perimeter of region} \approx 18 \text{ in} + 6.28 \text{ in} = 24.28 \text{ in} \]

So, the perimeter of the region is approximately 24.28 in.
In Concept 2: Surface Area and Volume, you will find a section on each of the following:

- Surface Area of a Rectangular Prism
- Volume of a Rectangular Prism
- Surface Area of a Cylinder
- Volume of a Cylinder
- Volume of a Cone
- Volume of a Sphere

A cube is a box with six square faces (sides). One cubic inch and one cubic centimeter cube are shown in Figure 38.

A rectangular prism is a box with 6 rectangles as faces (sides).

As shown in Figure 39, you can unfold a rectangular prism. This way you can see its 6 faces (sides).

Since each face (side) is a rectangle, you can find the area of a face (side) by using the formula: \( A = bh \).

To find the surface area of a rectangular prism:

- Find the area of each of the 6 faces (sides).
- Add these areas.
A rectangular prism, as shown in Figure 40, has three different measurements:

“length” “width” and “height.”

Figure 39

2 sides have area $l \times w$.
2 sides have area $l \times h$.
2 sides have area $w \times h$.

So, here’s another way to find the surface area of a rectangular prism:

- Identify the length, width, and height of the rectangular prism.
- Use the formula:

$$2(l \times w) + 2(l \times h) + 2(w \times h)$$

which is the same as

$$2 \times l \times w + 2 \times l \times h + 2 \times w \times h$$

23. A rectangular prism with dimensions 4 cm by 3 cm by 2 cm is shown in Figure 41. Find the surface area of the rectangular prism.

Figure 41

To find the surface area of the rectangular prism:

- Find the area of each of the 6 sides.

Find the area of the top rectangle.  
Area of top = $2 \text{ cm} \times 4 \text{ cm}$  
$= 8 \text{ cm}^2$

The area of the bottom rectangle is the same as the area of the top rectangle.  
Area of bottom = $8 \text{ cm}^2$

Find the area of the rectangle on the right.  
Area of right side = $2 \text{ cm} \times 3 \text{ cm}$  
$= 6 \text{ cm}^2$

The area of the rectangle on the left is the same as the area of the rectangle on the right.  
Area of left side = $6 \text{ cm}^2$
Find the area of the front rectangle.  

\[ \text{Area of front} = 3 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2 \]

The area of the back rectangle is the same as the area of the front rectangle.

\[ \text{Area of back} = 12 \text{ cm}^2 \]

• Add these 6 areas.

\[
\begin{align*}
8 \text{ cm}^2 \\
8 \text{ cm}^2 \\
6 \text{ cm}^2 \\
6 \text{ cm}^2 \\
12 \text{ cm}^2 \\
+ 12 \text{ cm}^2 \\
\hline
52 \text{ cm}^2
\end{align*}
\]

So, the surface area of the rectangular prism is 52 cm².

24. A rectangular prism with dimensions 
\[ 4 \frac{1}{4} \text{ in} \times 2 \frac{1}{2} \text{ in} \times 4 \frac{3}{4} \text{ in} \] is shown in Figure 42. Find the surface area of the rectangular prism.

To find the surface area of the rectangular prism:

• Identify the length, width and height of the rectangular prism.

\[
\begin{align*}
\text{length} &= 4 \frac{1}{4} \text{ in} \\
\text{width} &= 2 \frac{1}{2} \text{ in} \\
\text{height} &= 4 \frac{3}{4} \text{ in}
\end{align*}
\]

• Use the formula:

\[
\text{Surface Area} = 2 \times \text{L} \times \text{W} + 2 \times \text{L} \times \text{H} + 2 \times \text{W} \times \text{H}
\]

\[
\begin{align*}
&= 2 \times 4 \frac{1}{4} \text{ in} \times 2 \frac{1}{2} \text{ in} + 2 \times 4 \frac{1}{4} \text{ in} \times 4 \frac{3}{4} \text{ in} + 2 \times 2 \frac{1}{2} \text{ in} \times 4 \frac{3}{4} \text{ in} \\
&= 2 \times 4 \frac{1}{4} \text{ in} \times \frac{5}{2} \text{ in} + 2 \times 4 \frac{3}{4} \text{ in} \times \frac{19}{4} \text{ in} + 2 \times \frac{5}{2} \text{ in} \times \frac{19}{4} \text{ in} \\
&= \frac{85}{4} \text{ in}^2 + \frac{323}{8} \text{ in}^2 + \frac{95}{4} \text{ in}^2 \\
&= \frac{170}{8} \text{ in}^2 + \frac{323}{8} \text{ in}^2 + \frac{190}{8} \text{ in}^2 \\
&= \frac{683}{8} \text{ in}^2 \\
&= 85 \frac{3}{8} \text{ in}^2
\end{align*}
\]

So, the surface area of the rectangular prism is 85 \( \frac{3}{8} \) in².
Volume of a Rectangular Prism

Recall, a rectangle has two dimensions: length and width. See Figure 43.

![Figure 43](image)

A rectangular prism has three dimensions: length, width, and height. See Figure 44.

![Figure 44](image)

To find the volume of a rectangular prism:

- Multiply its dimensions.

$$V = \text{length} \times \text{width} \times \text{height}$$

As shown in Figure 45, the prism “rests” on its Base.

![Figure 45](image)

The area of the Base is length \times width.

$$B = lw$$

So, you can also write: $$V = Bh$$

Length is measured in “ruler” units such as inches, feet, centimeters, and so on.

![Figure 46](image)

Area is measured in square units such as square inches, square feet, and square centimeters.

![Figure 47](image)
Volume is measured in cubic units such as cubic inches, cubic feet, and cubic centimeters.

![Diagram of a cube with 1 cm³](image)

**Figure 48**

25. In Figure 49, find the volume of the rectangular prism.

![Diagram of a rectangular prism](image)

**Figure 49**

Here’s one way to find the volume of the rectangular prism:

- **Multiply its dimensions.**
  
  \[ V = \text{length} \times \text{width} \times \text{height} \]
  
  \[ = 5 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm} \]
  
  \[ = 60 \text{ cm}^3 \]

So, the volume of the rectangular prism is 60 cm³.

26. In Figure 50, find the volume of the rectangular prism.

![Diagram of a rectangular prism](image)

**Figure 50**

Here’s a way to find the volume of the rectangular prism:

- **Find the area of the base.**
  
  \[ B = \text{length} \times \text{width} \]
  
  \[ = 5.2 \text{ cm} \times 2.4 \text{ cm} \]
  
  \[ = 12.48 \text{ cm}^2 \]

- **Multiply the base by the height.**
  
  \[ V = B \times \text{height} \]
  
  \[ = 12.48 \text{ cm}^2 \times 3.1 \text{ cm} \]
  
  \[ = 38.688 \text{ cm}^3 \]

So, the volume of the rectangular prism is 38.688 cm³.
Surface Area of a Cylinder

You have seen how to find the surface area and volume of a rectangular prism. Now you will work with another 3–D figure called a cylinder.

A cylinder is a 3–D figure shaped like a can of soup.

As shown in Figure 51, the face the cylinder “rests” on is called the Base. The height is the perpendicular distance between the two circular faces.

To find the surface area of a cylinder:

- Find the area of each circle. (area of circular base and area of circular top)
- Find the area of the rectangle. (area of the “unrolled side” of the cylinder)
  - The height of the rectangle is the same as the height of the cylinder.
  - The base of the rectangle is the same as the circumference of the bottom circle.
- Add those areas.

If you picture a cylinder as shown in Figure 53, you can determine a formula to find the surface area of a cylinder.

The area of each circle is \(\pi r^2\).

The area of the rectangle is \(2\pi r \times h\).

So, you can write:

\[
SA = 2\pi r^2 + 2\pi rh
\]

27. Find the surface area of the cylinder shown in Figure 54.
Here’s one way to find the surface area of the cylinder:

- Find the area of each circle:
  The radius of each circle is 4 cm. 
  \[ A = \pi r^2 \]
  \[ = \pi \times r \times r \]
  \[ = \pi \times 4\text{ cm} \times 4\text{ cm} \]
  \[ = 16\pi \text{ cm}^2 \]

- Find the area of the rectangle:
  — The height of the rectangle is the same as the height of the cylinder. 
  — The base of the rectangle is the same as the circumference of the bottom circle.
  Area of rectangle: 
  \[ A = \text{base} \times \text{height} \]
  \[ = 2 \times \pi \times 4\text{ cm} \times 5\text{ cm} \]
  \[ = 40\pi \text{ cm}^2 \]

- Add these areas.
  \[ \text{SA} = 16\pi \text{ cm}^2 + 16\pi \text{ cm}^2 + 40\pi \text{ cm}^2 \]
  \[ = 72\pi \text{ cm}^2 \]

So, the surface area of the cylinder is \(72\pi \text{ cm}^2\).

28. Find the surface area of the cylinder shown in Figure 55.

Figure 55

Here’s a way to find the surface area of the cylinder:

- Use the formula.
  \[ \text{SA} = 2\pi r^2 + 2\pi rh \]
  \[ = 2 \times \pi \times 3\text{ in} \times 3\text{ in} + 2 \times \pi \times 3\text{ in} \times 10\text{ in} \]
  \[ = 18\pi \text{ in}^2 + 60\pi \text{ in}^2 \]
  \[ = 78\pi \text{ in}^2 \]

So, the surface area of the cylinder is \(78\pi \text{ in}^2\).

Example 28

To approximate the surface area of the cylinder, you can use 3.14 as an approximation for \(\pi\).

So, \(\text{SA} \approx 72 \times 3.14 \text{ cm}^2 \)
\[ = 226.08 \text{ cm}^2 \]
That is, the surface area of the cylinder is approximately \(226.08 \text{ cm}^2\).

Volume of a Cylinder

Now you will learn how to find the volume of a cylinder.

You have seen that to find the volume of the rectangular prism shown in Figure 56, you multiply:

\[ V = \text{area of Base} \times \text{height} \]

\[ V = lwh \]

Figure 56
To approximate the volume of the cylinder, you can use 3.14 as an approximation for \( \pi \). So,
\[
V = \pi r^2 \times h
\]
That is, the volume of the cylinder is approximately equal to 1256 in\(^3\).

Example 29

To find the volume of the cylinder shown in Figure 58.

Here’s one way to find the volume of the cylinder:

- Find the area of the circular base:
  
  The radius of the circle is 10 in.
  \[
  B = \pi r^2 = \pi \times 10 \times 10 = 100 \text{ in}^2
  \]

- Multiply the area of the base by the height.
  \[
  V = Bh = 100 \times 4 = 400 \text{ in}^3
  \]
  So, the volume of the cylinder is 400 \( \text{in}^3 \).

Example 30

The volume of a cylinder is \( 24\pi \) cubic feet. The area of the base is \( 4\pi \text{ ft}^2 \).

Find the height of the cylinder.

Here’s a way to find the height of the cylinder:

- Use the formula.
  \[
  V = B \times h
  \]
  Replace \( V \) by \( 24\pi \text{ ft}^3 \).
  \[
  24\pi \text{ ft}^3 = 4\pi \text{ ft}^2 \times h
  \]
  Replace \( B \) by \( 4\pi \text{ ft}^2 \).
  \[
  \frac{24\pi \text{ ft}^3}{4\pi \text{ ft}^2} = \frac{4\pi \text{ ft}^2 \times h}{4\pi \text{ ft}^2}
  \]
  \[
  6 \text{ ft} = h
  \]
  So, \( h \), the height of the cylinder is 6 ft.
Cones

Now you will find the volume of a cone.

As shown in Figure 59, the height, $h$, of the cone is the perpendicular distance from the vertex to the base. The base of the cone is a circle of radius $r$. The high point of the cone is called the vertex.

![Figure 59](vertex-diagram.png)

If a cone and a cylinder have the same height, $h$, and the same radius, $r$, then the volume, $v$, of the cone is one-third the volume of the cylinder. That is:

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

To find the volume of a cone:

- Use the formula: $V = \frac{1}{3} \pi r^2 h$.

**Caution!** To find the height of the cone pictured in Figure 60, do not measure along the curved side of the cone. Instead, measure the line segment that makes a right angle with the base and joins the base to the vertex.

![Figure 60](cone-diagram.png)

31. In Figure 61, find the volume of the cone.

Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

![Figure 61](cone-diagram-61.png)

**Example 31**

You may find these Examples useful while doing the homework for this section.
This is the volume of the cone. 

You can approximate the value by approximating $\pi$ with 3.14.

Divide 50.24 by 3. 

Round to 2 decimal places.

So, the volume of the cone is approximately 16.75 in³.

32. In Figure 62 find the volume of the cone.

Here’s a way to find the volume of the cone:

- Use the formula. 

So, the volume of the cone is 180 $\pi$ in³.

Spheres

You have learned how to find the volume of a cone. You can use what you know about the volume of a cone to find the volume of a sphere.

A sphere is a 3–D figure shaped like a ball.

As shown in Figure 63, the radius of a sphere is the distance from the center of the sphere to any point on the sphere.

If a cone has both its radius, $r$, and height, $h$, equal to the radius of the sphere, then the volume of the sphere is four times the volume of the cone. That is:

$$\text{Volume of sphere} = 4 \times \text{Volume of cone}$$

$$V = 4 \times \frac{1}{3} \times \pi \times r^2 \times r$$

$$V = \frac{4}{3} \pi r^3$$

The formula for the volume of a sphere can also be written as follows:

$$V = \frac{4\pi r^3}{3}$$

A hemisphere is one-half of a sphere.
The volume of a hemisphere is one-half the volume of a sphere with the same radius.

\[ \text{Volume of hemisphere} = \frac{1}{2} \times \frac{4}{3} \pi r^3. \]

**Figure 64**

Volume of hemisphere = \( \frac{1}{2} \times \frac{4}{3} \pi r^3 \).

33. Find the volume of the sphere shown in Figure 65. Then approximate the volume using 3.14 to approximate \( \pi \). Round your answer to two decimal places.

**Figure 65**

Here’s one way to find the volume of the sphere:

- **Use the formula.**
  \[ V = \frac{4}{3} \pi r^3 \]

- **Replace \( r \) by 6 in.**
  \[ V = \frac{4}{3} \pi \times 6 \text{ in} \times 6 \text{ in} \times 6 \text{ in} \]
  \[ = \frac{4}{3} \times \pi \times 216 \text{ in}^3 \]

- **This is the volume of the sphere.**
  \[ = 864 \pi \text{ in}^3 \]

- **Approximate \( \pi \) with 3.14.**
  \[ V = \frac{864(3.14)}{3} \text{ in}^3 \]

- **Divide 2712.96 by 3.**
  \[ = \frac{2712.96}{3} \text{ in}^3 \]
  \[ = 904.32 \text{ in}^3 \]

So, the volume of the sphere is approximately 904.32 \( \text{in}^3 \).

34. Find the volume of the hemisphere shown in Figure 66.

**Figure 66**

Here’s one way to find the volume of the hemisphere:

- **Find half the volume of the sphere.**
  \[ V = \frac{1}{2} \times \frac{4}{3} \pi r^3 \]

- **Replace \( r \) by 3 cm.**
  \[ = \frac{1}{2} \times \frac{4}{3} \pi \times 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} \]

- **This is the volume of the sphere.**
  \[ = 18\pi \text{ cm}^3 \]

So, the volume of the sphere is 18\( \pi \text{ cm}^3 \).
Example 35 35. In Figure 67, find the volume of the 3-D figure (a rectangular prism with a hole bored through it). Use 3.14 to approximate \( \pi \).

![Figure 67](image)

Here’s one way to find the volume of the figure:

- **Find the volume of the rectangular prism.**
  
  \[
  V = lwh
  \]
  
  Replace \( l \) by 7 cm, \( w \) by 5 cm, \( h \) by 5 cm.
  
  This is the exact volume.
  
  \[
  = 7 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}
  = 175 \text{ cm}^3
  \]

- **Find the volume of the cylinder.**
  
  Approximate \( \pi \) with 3.14.
  
  \[
  V = \pi r^2h
  \]
  
  Replace \( r \) by 2 cm.
  
  \[
  = 3.14 \times 2 \text{ cm} \times 2 \text{ cm} \times 7 \text{ cm}
  = 87.92 \text{ cm}^3
  \]

- **Subtract the volume of the cylinder from the volume of the rectangular prism.**
  
  \[
  = 175 \text{ cm}^3 - 87.92 \text{ cm}^3
  = 87.08 \text{ cm}^3
  \]

So, the volume of the figure is approximately 87.08 cm\(^3\).

Example 36 36. Find the volume of the 3-D figure shown in Figure 68. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

![Figure 68](image)

Here’s one way to find the volume of the figure:

- **Find the volume of the hemisphere.**
  
  \[
  V = \frac{1}{2} \times \frac{4}{3} \pi r^3
  \]
  
  Replace \( r \) by 1 ft.
  
  Approximate \( \pi \) with 3.14.
  
  \[
  = \frac{1}{2} \times \frac{4}{3} \pi \times 1 \text{ ft} \times 1 \text{ ft} \times 1 \text{ ft}
  = 2.09 \text{ ft}^3
  \]

- **Find the volume of the cylinder.**
  
  \[
  V = \pi r^2h
  \]
  
  Replace \( r \) by 1 ft.
  
  Approximate \( \pi \) with 3.14.
  
  \[
  = 3.14 \times 1 \text{ ft} \times 1 \text{ ft} \times 3 \text{ ft}
  = 9.42 \text{ ft}^3
  \]

- **Add the volume of the hemisphere to the volume of the cylinder.**
  
  \[
  = 2.09 \text{ ft}^3 + 9.42 \text{ ft}^3
  = 11.51 \text{ ft}^3
  \]

So, the volume of the figure is approximately 11.51 ft\(^3\).
**Investigation 1: Don’t Fence Me In**

What is the greatest possible rectangular area you can enclose with 30 feet of fencing? To help you answer this question, complete the following tables and answer the corresponding questions.

1. Consider rectangles whose length and width are whole numbers greater than zero.

   a. Use the table below to list the length, width, and area of all possible rectangles with perimeter 30 feet.

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Area</th>
<th>Perimeter = 30 ft</th>
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   b. Which rectangle has the greatest area?

   c. Which rectangle has the least area?

2. Now, use decimal numbers, such as 4.5 feet, for the length and width.

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Area</th>
<th>Perimeter = 30 ft</th>
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   a. Find a rectangle with area greater than the one you found in part (1.b).

   b. Find a rectangle with area less than the one you found in part (1.c).
3.  a. What is the length and the width of the rectangle with the greatest area that you can find?

b. Do you think there is a rectangle whose perimeter is 30 and which has the greatest possible area, or is there always some rectangle with area greater than the one you find? Justify your answer.

4.  a. What is the length and the width of the rectangle with the least area that you can find?

b. Do you think there is a rectangle whose perimeter is 30 and which has the least possible area, or is there always some rectangle with area less than the one you find? Justify your answer.

5. To test your ideas, repeat this exploration by examining rectangles with perimeter 36 feet.

**Investigation 2: Packaging Products**

What is the least possible surface area of a rectangular box (with a top) that holds 240 cubic inches? To help you answer this question, complete the following tables and answer the corresponding questions.

1. Consider rectangular boxes whose length, width, and height are whole numbers greater than zero.
   a. Use the table on the next page to list the length, width, height and surface area of all possible rectangular boxes with volume 240 cubic inches.
   b. Which rectangular box has the greatest surface area?
   c. Which rectangular box has the least surface area?
2. Now, use decimal numbers, such as 4.5 inches, for the length, width, and height.
   a. Find a rectangular box with surface area greater than the one you found in part (1.b).
   b. Find a rectangular box with surface area less than the one you found in part (1.c).

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
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<th>Area</th>
<th>Volume = 240 cu in</th>
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3. a. What are the length, width, and height of the rectangular box with the greatest surface area you can find?

   b. Do you think there is a rectangular box whose volume is 240 cu in and which has the greatest possible surface area, or is there always some rectangular box with surface area greater than the one you find? Justify your answer.

4. a. What are the length, width, and height of the rectangular box with the least surface area you can find?

   b. Do you think there is a rectangular box whose volume is 240 cu in and which has the least possible surface area, or is there always some rectangular box with surface area less than the one you find? Justify your answer.

5. To test your ideas, repeat this exploration by examining rectangular boxes with 300 cubic inches.

**Investigation 3: Why π?**

For this investigation, you will need a long piece of string, a ruler, and at least 12 objects whose cross-sections are circles (different size jar lids, cans, etc.).

You have learned that \( C \), the circumference of a circle, is \( \pi \) times \( d \), the diameter of the circle.

\[ C = \pi \times d \]

If you divide both sides of this equation by \( d \), you can see that \( \pi \) is the ratio of the circumference to the diameter of the circle.

\[ \pi = \frac{C}{d} \]

This shows that the ratio of circumference to diameter is the same for circles of all sizes.
In this investigation, you will test this result.

1. Use the piece of string and the ruler to carefully measure the circumference and diameter of at least 12 “circular objects.” Record the data in the table below.

<table>
<thead>
<tr>
<th>Circumference (C)</th>
<th>Diameter (d)</th>
<th>Ratio C/d</th>
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2. For each circle, calculate the ratio \( \frac{C}{d} \). Record the results in the table in part (1).

3. Plot each ratio on the following number line.

4. Do your results support the claim that the ratio \( \frac{C}{d} \) is the same for circles of all sizes? Explain.

5. a. How many of your ratios are greater than \( \pi \)? (You can use 3.14 to approximate \( \pi \).)

   b. How many of your ratios are less than \( \pi \)? (You can use 3.14 to approximate \( \pi \).)

   c. Are your ratios distributed equally so that half of them are less than \( \pi \) and half of them are greater than \( \pi \)?

   d. Explain your answer in (c).
6. a. Compare your ratios for large circles with your ratios for small circles. Are the ratios for large circles greater than the ratios for small circles?

b. Are the ratios for large circles less than the ratios for small circles?

c. Do your answers in parts (a) and (b) support the fact that the ratio \( \frac{C}{d} \) is the same for circles of all sizes?

7. Which of your ratios are closer to the value of \( \pi \): the ratios for small circles or the ratios for large circles? Explain. (Hint: Every measurement is an approximation. Do you think measurement errors “count” more in a large measurement or in a small measurement?)

8. a. Calculate the average (mean) of all your ratios:
   i. In your table, find the sum of all the ratios. ___________________
   ii. Divide the result by the number of ratios. (This is the average [mean] of all your ratios.) ___________________

b. Is this average closer to the value of \( \pi \) than most of your measured ratios? Explain.
Concept 1: Area and Perimeter

Perimeter of a Polygon

For help working these types of problems, go back to Examples 1–3 in the Explain section of this lesson.

1. Find the perimeter of the polygon.

2. Find the perimeter of the pentagon.

3. Find the perimeter of the triangle.

4. Find the perimeter of the triangle.

5. Find the perimeter of the quadrilateral.

6. Find the perimeter of the quadrilateral.

7. Find the perimeter of the trapezoid.

8. Find the perimeter of the trapezoid.

9. If the perimeter of the pentagon shown is 35 ft, find x.

10. If the perimeter of the pentagon shown is 53 meters, find x.
11. If the perimeter of the quadrilateral shown is $43\frac{1}{2}$ inches, find $x$.

12. If the perimeter of the quadrilateral shown is 26.3 cm, find $x$.

13. If the perimeter of the octagon shown is 25 miles, find $x$.

14. If the perimeter of the octagon shown is 45 yards, find $x$.

15. If the perimeter of the polygon shown is 37 meters, find $x$.

16. If the perimeter of the pentagon shown is 27.2 feet, find $x$.

17. A rancher is building a fence around his new pig pen. The corral is rectangular in shape with a length of 100 feet and a width of 80 feet. How many feet of fence must he build in order to completely enclose the corral?

18. A rancher is building a fence around his new cow pen. The corral is rectangular in shape with a length of 85.3 feet and a width of 47.6 feet. How many feet of fence must he build in order to completely enclose the corral?

19. The police are taping off a crime scene with yellow tape. How much tape will they have to use to rope off a rectangular region that is 8 yards long and 5 yards wide?

20. The police are taping off a crime scene with yellow tape. How much tape will they have to use to rope off a rectangular region that is 13.7 feet long and 9.9 feet wide?

21. Javier is installing lights around a park picnic enclosure. He wishes to run a wire completely around the 7 walls. The lengths of the walls are 7 feet, 10 feet, 12 feet, 13 feet, 9 feet, 11 feet, and 14 feet. What length of wire should he cut?

22. Josie is installing lights around a park picnic enclosure. He wishes to run a wire completely around the 5 walls. The lengths of the walls are $6\frac{3}{4}$ feet, $5\frac{5}{8}$ feet, $8\frac{1}{2}$ feet, $9\frac{5}{8}$ feet, and $7\frac{3}{8}$ feet. What length of wire should she cut?

23. Nadia is rewallpapering her house. She wishes to run a foot-wide border all the way around her dining room. If the walls in the dining room are 8.6, 9.5, 12.3, and 13.4 feet wide, how many feet of wallpaper border should she buy to have just the right amount of wallpaper?

24. Ken is rewallpapering his house. He wishes to run a foot-wide border all the way around his living room. If the walls in the living room are $14\frac{3}{4}$, $10\frac{1}{2}$, $12\frac{1}{4}$, and 15 feet wide, how many feet of wallpaper border should he buy to have just the right amount of wallpaper?
Area of a Rectangle

For help working these types of problems, go back to Examples 4–6 in the Explain section of this lesson.

25. Find the area of the rectangle.

26. Find the area of the rectangle.

27. Find the area of the rectangle.

28. Find the area of the rectangle.

29. Find the area of the rectangle.

30. Find the area of the rectangle.

31. Find the area of the rectangle.

32. Find the area of the rectangle.

33. If a rectangle is 8 feet long and 5 feet wide, find its area.

34. If a rectangle is 10.6 centimeters long and 7.4 centimeters wide, find its area.

35. If a rectangle is 6 inches long and 4 inches wide, find its area.

36. If a rectangle is 5 meters long and 3 meters wide, find its area.

37. If the area of the rectangle shown is 45 square inches, find \( x \).

38. If the area of the rectangle shown is 56 square centimeters, find \( x \).
39. If the area of the rectangle shown is 37.12 square meters, find $x$.

40. If the area of the rectangle shown is $8\frac{16}{25}$ square miles, find $x$.

41. The owners of a sports arena wish to repave its rectangular parking lot. If the parking lot is 545 feet long and 423 feet wide, how many square feet of surface must they repave?

42. The owners of a sports arena wish to repave its rectangular parking lot. If the parking lot is 210 yards long and 147 yards wide, how many square feet of surface must they repave? (Note: There are 9 square feet per square yard.)

43. The field maintenance crew at a university is planning to re-sod the football field inside the stadium. If the field is 160 yards long and 60 yards wide, how many square feet of sod should the maintenance workers order? (Note: There are 9 square feet per 1 square yard.)

44. The field maintenance crew at a university is planning to re-sod the football field inside the stadium. If the field is 140 yards long and 55 yards wide, how many square feet of sod should the maintenance workers order? (Note: There are 9 square feet per 1 square yard.)

45. Amir is recarpeting his rectangular bedroom. If his bedroom is $12\frac{3}{4}$ feet long and $10\frac{1}{4}$ feet wide, how many square feet of carpet does he need?

46. Vladimir is recarpeting his rectangular bedroom. If his bedroom is $10\frac{3}{4}$ feet long and $8\frac{1}{4}$ feet wide, how many square feet of carpet does he need?

47. The maintenance crew at a gymnasium cover the gym’s floor with a tarp to protect it during an upcoming computer convention. If the floor is 196 feet long and 102 feet wide, how many square feet of tarp should they purchase to completely cover the floor?

48. The Red Cross needs to repaint the large red cross on the outside of its headquarters. If every edge of the cross is 5 feet long, how many square feet will they have to paint to give the entire cross one new coat of paint? (See the figure.)

**Parallelograms**

For help working these types of problems, go back to Examples 7–9 in the Explain section of this lesson.

49. Find the area of the parallelogram.

50. Find the area of the parallelogram.

51. Find the area of the parallelogram.
52. Find the area of the parallelogram.

53. Find the area of the parallelogram.

54. Find the area of the parallelogram.

55. If the area of the parallelogram shown is 20 square feet, find $x$.

56. If the area of the parallelogram shown is 32 square meters, find $x$.

57. If the area of the parallelogram shown is 64.6 square inches, find $x$.

58. If the area of the parallelogram shown is 66.74 square miles, find $x$.

59. If the area of the parallelogram shown is $38\frac{16}{25}$ square feet, find $x$.

60. If the area of the parallelogram shown is $17\frac{9}{25}$ square centimeters, find $x$.

61. If the area of the parallelogram shown is 28 square feet, find the height of the parallelogram.

62. If the area of the parallelogram shown is 36 square inches, find the height of the parallelogram.

63. If the area of the parallelogram shown is 120 square centimeters, find the height of the parallelogram.

64. If the area of the parallelogram shown is 204 square feet, find the height of the parallelogram.
65. Roberto’s dad is building his son a sandbox in their backyard. Roberto insists that his sandbox be shaped like a parallelogram, even though his father prefers the classic rectangular shape. The parallelogram-shaped sandbox shown has 2 edges that are 8 feet long, two edges that are 5 feet long, and a height of 4 feet. Find the area of the sandbox.

66. Edmundo’s dad is building his son a sandbox in their backyard. Edmundo insists that his sandbox be shaped like a parallelogram, even though his father prefers the classic rectangular shape. The parallelogram-shaped sandbox shown has 2 edges that are 10 feet long, two edges that are 7 feet long, and a height of 6 feet. Find the area of the sandbox.

67. Kevin is trying to find the square footage of the floor of his parallelogram-shaped sun room. The room has 2 walls that are 12.3 feet long and 2 walls that are 8.9 feet long, and the distance between the longer walls is 7.4 feet. Find the area of the sun room’s floor.

68. Wael is trying to find the square footage of the floor of his parallelogram-shaped reading room. The room as shown has 2 walls that are 11.4 feet long and 2 walls that are 8.7 feet long, and the distance between the longer walls is 6.8 feet. Find the area of the floor of Wael’s reading room.

69. Archaeologists have uncovered a large mosaic. In order to catalog the work, they must know its area. Each tile is in the shape of a parallelogram. If the mosaic is made up of 6 identical tiles, find its area. The height of each parallelogram is 4.5 feet and the longest edge is 9.3 feet long as shown.

70. Archaeologists have uncovered a large mosaic. In order to catalog the work, they must know its area. Each tile is in the shape of a parallelogram. If the mosaic is made up of 6 identical tiles, find its area. The height of each parallelogram shown is $3 \frac{1}{3}$ feet and the longest edge is $6 \frac{2}{3}$ feet long as shown.

71. Ebony has to create a new playground game for a project in her P.E. class. She decides to alter the popular game of four-square, creating a new game she calls four-parallelogram. As part of the project, her teacher requires that the students calculate the area of their playing field. Ebony’s playing field is pictured. Find the total area of the playing field. Each of the four parallelograms has the same area.

72. Courtney has to create a new playground game for a project in her P.E. class. She decides to alter the popular game of four-square, creating a new game she calls four-parallelogram. As part of the project, her teacher requires that the students calculate the area of their playing field. Courtney’s playing field is pictured. Find the area of the playing field. Each of the four parallelograms has the same area.
**Triangles**

For help working these types of problems, go back to Examples 10–12 in the Explain section of this lesson.

73. Find the area of the triangle.

74. Find the area of the triangle.

75. Find the area of the triangle.

76. Find the area of the triangle.

77. Find the area of the triangle.

78. Find the area of the triangle.

79. Find the area of the triangle.

80. Find the area of the triangle.

81. Find the area of the triangle.

82. Find the area of the triangle.

83. The area of the triangle shown is 30 square feet. Find $x$.

84. The area of the triangle shown is 22 square inches. Find $x$.

85. The area of the triangle shown is 37.8 square meters. Find $x$. Round your answer to two decimal places.
86. The area of the triangle shown is 5.035 square inches. Find $x$.

87. The area of the triangle shown is $3053 \frac{2}{25}$ square meters. Find $x$.

88. The area of the triangle shown is $1207 \frac{9}{25}$ square meters. Find $x$.

89. Kevin wishes to repaint his grandfather’s wheelchair ramp. The sides of the ramp are right triangles. The length of the ramp shown, along the sloped surface is 15.3 feet, the base of the ramp is 15 feet, and the height of the ramp is 3 feet. What is the area of one side of the ramp?

90. Jan wants to repaint her mother’s wheelchair ramp as a Mother’s Day surprise. The sides of the ramp are right triangles. The length of the ramp shown, along the sloped surface is 12.5 feet, the base of the ramp is 12 feet, and the height of the ramp is 3.5 feet. What is the area of the side of the ramp?

91. Lloyd is building a skateboard ramp out of plywood. To build the sides of the ramp, he will cut right triangles out of the plywood. Lloyd wants the ramp to be 3.4 feet high and its base to be 5.3 feet long. How many square feet of plywood does he need to build one side of the ramp?

92. Jude is building a skateboard ramp out of plywood. To build the sides of the ramp, he will cut right triangles out of the plywood. Jude wants the ramp to be 3.7 feet high and its base to be 6.2 feet long. How many square feet of plywood does he need to build one side of the ramp?

93. Mr. Abolnikov built a deck off the back of his house. The deck is triangular and has area of 420 square feet. One edge of the deck runs along the back of the house which is 70 feet long. How far does the deck extend from his house at its farthest point?

94. Mr. Tolstoy is built a deck off the back of his house. The deck is triangular and has area 640 square feet. One edge of the deck runs along the back of the house, which is 40 feet wide. How far should the deck extend from his house at its farthest point?

95. A pennant manufacturer produces pennants for all occasions. Their most popular item is the classic triangular felt pennant. If this pennant is 4.1 feet long from the center of its base to its tip and its base is 2.3 feet long, how many square feet of felt are needed to make the pennant?

96. A pennant manufacturer produces pennants for all occasions. Their most popular item is the classic triangular felt pennant. If this pennant is 3.4 feet long from the center of its base to its tip and its base is 1.8 feet long, how many square feet of felt are needed to make the pennant?

**Trapezoids**

For help working these types of problems, go back to Examples 13–15 in the Explain section of this lesson.

97. Find the area of the trapezoid.

98. Find the area of the trapezoid.
99. Find the area of the trapezoid.

100. Find the area of the trapezoid.

101. Find the area of the trapezoid.

102. Find the area of the trapezoid.

103. Find the area of the trapezoid.

104. Find the area of the trapezoid.

105. Find the area of the trapezoid.

106. The area of the trapezoid shown is 10 square feet. Find $x$.

107. The area of the trapezoid shown is 78 square centimeters. Find $x$.

108. The area of the trapezoid shown is 61.75 square feet. Find $x$.

109. The area of the trapezoid shown is 51 square inches. Find $x$. 

\[ \text{Area} = \frac{1}{2} \times (b_1 + b_2) \times h \]
110. The area of the trapezoid shown is 13.5 square centimeters. 
Find x.

![Trapezoid Diagram]

111. The area of the trapezoid shown is 44.25 square meters. 
Find x.

![Trapezoid Diagram]

112. The area of the trapezoid shown is 6335 square feet. 
Find x.

![Trapezoid Diagram]

113. John has decided to sell his house. Before he can list the house with a real-estate agent, he must know the square footage of his house. One of the rooms in John’s house is shaped like a trapezoid. The dimensions of the room are shown. What is the area of this room?

![Room Diagram]

114. John has decided to sell his house. Before he can list the house with a real-estate agent, he must know the square footage of his house. One of the rooms in John’s house is shaped like a trapezoid. The dimensions of the room are shown. What is the area of this room?

![Room Diagram]

115. To prevent speeding, many stores have placed speed bumps throughout their parking lots. If these speed bumps are viewed from the side, their cross-sections are trapezoids. If a speed bump is 5 inches tall, has a base which is 17 inches long, and its top is 13 inches long, what is the area of the speed bump’s cross-section? It may be helpful to draw and label a sketch.

116. To prevent speeding, many stores have placed speed bumps throughout their parking lots. If these speed bumps are viewed from the side, their cross-sections are trapezoids. If a speed bump is 4.5 inches tall, has a base which is 17 inches long, and its top is 12.7 inches long, what is the area of the speed bump’s cross-section? It may be helpful to draw and label a sketch.

117. Felipe custom builds picture frames for his clients. One client has a rectangular picture that is 11 inches wide and 14 inches long that he wishes to have framed. He wants the frame to be 20 inches long and 15 inches wide along its outside edges. On its inside edges, the frame is to be flush with the picture. He plans to make the frame using 4 pieces of wood, each shaped like a trapezoid. What is the total area of wood that Felipe must use to build the frame to his client’s specifications? (Draw and label a sketch.)

118. Lito custom builds picture frames for his clients. One client has a rectangular picture that is 8 inches wide and 10 inches long that he wishes to have framed. He wants the frame to be 16 inches long and 14 inches wide along its outside edges. On its inside edges, the frame is to be flush with the picture. He plans to make the frame using 4 pieces of wood, each shaped like a trapezoid. What is the total area of wood that Lito must use to build the frame to his client’s specifications? (Draw and label a sketch.)

119. Malcolm is building a wooden keepsake box for his mom’s Mother’s Day present. The sides of the box are trapezoids. The lid of the box, as well as the base of the box, is a square. The area of the base is 100 square inches. The area of the lid is 64 square inches. What is the area of one side of the box if the side is 3 inches tall along its surface?

120. Marvin is building a wooden keepsake box for his mom’s Mother’s Day present. The sides of the box are trapezoids. The lid of the box, as well as the base of the box, is a square. The area of the base is 81 square inches. The area of the lid is 64 square inches. What is the area of one side of the box if the side is 4 inches tall along its surface?
Circles

For help working these types of problems, go back to Examples 16–22 in the Explain section of this lesson.

121. The radius of a circle is 8 inches. Find its area.

122. The radius of a circle is 14 centimeters. Find its area.

123. The radius of a circle is 7.6 centimeters. Find its area.
   Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

124. The radius of a circle is 9.2 inches. Find its area.
   Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

125. The radius of a circle is \( 2 \frac{2}{3} \) feet. Find its area. Use \( \frac{22}{7} \) to approximate \( \pi \).

126. The radius of a circle is \( 6 \frac{1}{8} \) m. Find its area. Use \( \frac{22}{7} \) to approximate \( \pi \).

127. The diameter of a circle is 4 feet. Find its circumference.

128. The diameter of a circle is 9 centimeters. Find its circumference.

129. The radius of a circle is 7 centimeters. Find its circumference.

130. The radius of a circle is 18 inches. Find its circumference.

131. The diameter of a circle is 15.4 meters. Find its circumference. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

132. The radius of a circle is \( 3 \frac{1}{2} \) centimeters. Find its circumference. Use \( \frac{22}{7} \) to approximate \( \pi \).

133. Find the perimeter and the area of the object.

134. Find the perimeter and the area of the object.

135. Find the area of the shaded region.
   Use 3.14 to approximate \( \pi \).

136. Find the perimeter and the area of the object. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

137. Police decide to rope off a crime scene using yellow crime scene tape. If the tape is to run completely around a circular cul-de-sac, what length of tape should be measured from the roll of tape before the cut is made? The radius of the cul-de-sac is 30 feet. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

138. Police decide to rope off a crime scene using yellow crime scene tape. If the tape is to run completely around a circular cul-de-sac, what length of tape should be measured from the roll of tape before the cut is made? The diameter of the cul-de-sac is 52 feet. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

139. A circular helicopter landing pad on the roof of a hospital needs to be repainted. The radius of the helicopter pad is 17 feet. Paint costs $6.49 per gallon, including tax, and can only be purchased in gallon buckets. If one gallon of paint can cover 100 square feet with a single coat of paint, how much will it cost to give the entire helicopter pad 2 new coats of paint? Don’t forget that paint must be purchased by the gallon.
140. The circular helicopter landing pad on the roof of a hospital needs to be repainted. The radius of the helicopter pad is 15 feet. Paint costs $4.98 per gallon, including tax, and can only be purchased in gallon buckets. If one gallon of paint can cover 125 square feet with a single coat of paint, how much will it cost to give the entire helicopter pad 2 new coats of paint? Don’t forget that paint must be purchased by the gallon.

141. The diameter of the base of a pie tin is 9 inches. The diameter of the top of the pie tin is 12 inches. How many square inches of crust need to be prepared in order to create a crust on the bottom and top but not on the sides of a pie made in this tin?

142. The diameter of the base of a pie tin is 8 inches. The diameter of the top of the pie tin is 10 inches. How many square inches of crust need to be prepared in order to create a crust on the bottom and top but not on the sides of a pie made in this tin?

143. A wheel with a radius of 10.5 inches is coated with wet paint and allowed to roll long enough so that the point on the wheel that was initially at the top returns to the top. (In other words, the wheel makes one complete revolution and is then stopped.) As it rolls, the wheel paints a line upon the ground. How long is this line of paint? (Hint: think about the relationship between circumference and distance traveled.) Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

144. A wheel with a radius of 13 inches is coated with wet paint and allowed to roll long enough so that the point on the wheel that was initially at the top returns to the top. (In other words, the wheel makes one complete revolution and is then stopped.) As it rolls, the wheel paints a line upon the ground. How long is this line of paint? (Hint: think about the relationship between circumference and distance traveled.) Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

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**Concept 2: Surface Area and Volume**

**Surface Area of a Rectangular Prism**

For help working these types of problems, go back to Examples 23–24 in the Explain section of this lesson.

145. A rectangular prism is 10 inches long, 8 inches wide, and 9 inches tall. What is its surface area?

146. A rectangular prism is 12 inches long, 9 inches wide, and 7 inches tall. What is its surface area?

147. A rectangular prism is 15 centimeters long, 12 centimeters wide, and 9 centimeters tall. What is its surface area?

148. A rectangular prism is 14 centimeters long, 13 centimeters wide, and 11 centimeters tall. What is its surface area?

149. A rectangular prism is 8.5 inches long, 6.3 inches wide, and 5.2 inches tall. What is its surface area?

150. A rectangular prism is 6.7 inches long, 5.8 inches wide, and 5.4 inches tall. What is its surface area?

151. A rectangular prism is 12.8 centimeters long, 10.5 centimeters wide, and 7.3 centimeters tall. What is its surface area?

152. A rectangular prism is 3.5 meters long, 2.3 meters wide, and 1.6 meters tall. What is its surface area?

153. A rectangular prism is 5 $\frac{1}{4}$ feet long, 4 $\frac{3}{4}$ feet wide, and 3 $\frac{1}{2}$ feet tall. What is its surface area?

154. A rectangular prism is 8 $\frac{1}{4}$ inches long, 6 $\frac{1}{4}$ inches wide, and 5 $\frac{3}{4}$ inches tall. What is its surface area?

155. A rectangular prism is 15 $\frac{1}{4}$ centimeters long, 14 $\frac{1}{2}$ centimeters wide, and 11 centimeters tall. What is its surface area?

156. A rectangular prism is 19 $\frac{1}{4}$ centimeters long, 14 $\frac{1}{2}$ centimeters wide, and 7 centimeters tall. What is its surface area?

157. A cube has a surface area of 384 square inches. Find the length of an edge. (Remember that all the edges of a cube are the same length.)

158. A cube has a surface area of 1350 square inches. Find the length of an edge. (Remember that all the edges of a cube are the same length.)
159. Find the surface area of the rectangular prism.

160. Find the surface area of the rectangular prism.

161. A building is 24 stories tall. The base and roof of the building are squares that each have an area of 6400 square feet. How many gallons of paint are required to paint the outside of the building with one coat of paint if one gallon of paint covers 200 square feet with a single coat? One story is 10 feet tall. (Note: The roof is to be included in the painting but not the base.)

162. A building is 19 stories tall. The base and roof of the building are squares that each have an area of 8100 square feet. How many gallons of paint are required to paint the outside of the building with one coat of paint if one gallon of paint covers 180 square feet with a single coat? One story is 10 feet tall. (Note: The roof is to be included in the painting but not the base.)

163. Omar is wrapping a gift. He has placed the gift in a box shaped like a rectangular prism. The box is 10 inches tall, 12 inches long, and 8 inches wide. How many square inches of wrapping paper are needed to cover the entire surface of the box with no overlap?

164. Sally is wrapping a gift. She has placed the gift in a box shaped like a rectangular prism. The box is 9 inches tall, 11 inches long, and 7 inches wide. How many square inches of wrapping paper are needed to cover the entire surface of the box with no overlap?

165. The owners of an amusement park are remodeling the House of Mirrors. They need to know how many square meters of mirrored glass to purchase for the project. The main room is 15 meters long, 4 meters tall, and 9 meters wide. If the room is to have mirrors completely covering all four walls, the ceiling, and the floor, how many square meters of mirrored glass are needed to complete the main room?

166. The owners of an amusement park are remodeling the House of Mirrors. They need to know how many square meters of mirrored glass to purchase for the project. The main room is 12.5 meters long, 3.8 meters tall, and 7.3 meters wide. If the room is to have mirrors completely covering all four walls, the ceiling, and the floor, how many square meters of mirrored glass are needed to complete the main room?

167. Gregor has been contracted to renovate all of the rooms in a hospital. Part of this project entails padding the walls and ceilings of 10 rooms. Each room is 9 feet long, 7 feet tall, and 8 ft wide. If Gregor charges $1.58 for each square foot of padding he installs, how much will the hospital owe Gregor to pad the 10 rooms? (Ignore the thickness of the padding.)

168. Shavannah has been contracted to renovate all of the rooms in a hospital. Part of this project entails padding the walls and ceilings of 8 rooms. Each room is 10 ft long, 7 ft tall, and 8 ft wide. If Shavannah charges $1.83 for each square foot of padding she installs, how much will the hospital owe Shavannah to pad the 8 rooms? (Ignore the thickness of the padding.)

**Volume of a Rectangular Prism**

For help working these types of problems, go back to Examples 25–26 in the Explain section of this lesson.

169. A rectangular prism is 10 inches long, 8 inches wide, and 9 inches tall. What is its volume?

170. A rectangular prism is 12 inches long, 9 inches wide, and 7 inches tall. What is its volume?
171. A rectangular prism is 15 centimeters long, 12 centimeters wide, and 9 centimeters tall. What is its volume?

172. A rectangular prism is 14 centimeters long, 13 centimeters wide, and 11 centimeters tall. What is its volume?

173. A rectangular prism is 8.5 feet long, 6.3 feet wide, and 5.2 feet tall. What is its volume?

174. A rectangular prism is 6.7 feet long, 5.8 feet wide, and 5.4 feet tall. What is its volume?

175. A rectangular prism is 12.8 meters long, 10.5 meters wide, and 7.3 meters tall. What is its volume?

176. A rectangular prism is 3.5 meters long, 2.3 meters wide, and 1.6 meters tall. What is its volume?

177. A rectangular prism is 5 1/4 inches long, 4 3/4 inches wide, and 3 1/2 inches tall. What is its volume?

178. A rectangular prism is 8 1/4 inches long, 6 1/4 inches wide, and 5 3/4 inches tall. What is its volume?

179. A rectangular prism is 15 1/4 centimeters long, 14 1/2 centimeters wide, and 11 centimeters tall. What is its volume?

180. A rectangular prism is 19 1/4 centimeters long, 14 1/2 centimeters wide, and 7 centimeters tall. What is its volume?

181. A cube has a volume of 3375 cubic inches. Find the length of an edge. (Remember that all the edges of a cube are the same length.)

182. A cube has a volume of 729 cubic inches. Find the length of an edge. (Remember that all the edges of a cube are the same length.)

183. Find the volume of the rectangular prism.

184. Find the volume of the rectangular prism.

185. A high school is refilling its swimming pool. The rectangular shaped pool has a fixed depth of 7 feet and is 75 feet long and 50 feet wide. If water costs $0.02 per cubic foot, how much will it cost to completely fill the swimming pool?

186. A high school is refilling its swimming pool. The rectangular shaped pool has a fixed depth of 6.5 feet and is 80 feet long and 55 feet wide. If water costs $0.02 per cubic foot, how much will it cost to completely fill the swimming pool?

187. Melinda is donating plasma. When her donation is complete, Melinda’s plasma fills a container in the shape of a rectangular prism. The area of the container’s base is 78 square centimeters and the container is 4 centimeters tall. If 1 liter equals 1000 cubic centimeters, how many liters of plasma did Melinda donate?

188. Malik is donating plasma. When his donation is complete, Malik’s plasma fills a container in the shape of a rectangular prism. The area of the container’s base is 67 square centimeters and the container is 5 centimeters tall. If 1 liter equals 1000 cubic centimeters, how many liters of plasma did Malik donate?
189. The volume of an iceberg is unknown. Scientists wish to
determine its volume, so they place the iceberg in a deep
tank that is 70 feet long and 65 feet wide. When the iceberg
melts completely, the water level in the tank rises to 90 feet
above the floor of the tank. Assuming that water does not
expand when it freezes (or contract when it melts), what was
the volume of the original iceberg?

190. The volume of an iceberg is unknown. Scientists wish to
determine its volume, so they place the iceberg in a deep
tank that is 65 feet long and 62 feet wide. When the iceberg
melts completely, the water level in the tank rises to 76 feet
above the floor of the tank. Assuming that water does not
expand when it freezes (or contract when it melts), what was
the volume of the original iceberg?

191. Ivan is building a trunk to hold all of the items he plans to
take on an upcoming trip to the United States. He figures that
a trunk with a volume of 26 cubic feet should be large enough
to hold his belongings. If he builds the trunk so that it is 4.5
feet long and 2 feet wide, how tall should he make the trunk
so that it has the desired volume?

192. Macario is building a trunk to hold all of the items he plans
to take on an upcoming trip to Europe. He figures that a
trunk with a volume of 24 cubic feet should be large enough
to hold his belongings. If he builds the trunk so that it is
4 feet long and 2 feet wide, how tall should he make the
trunk so that it has the desired volume?

**Surface Area of a Cylinder**

For help working these types of problems, go back to Examples
27–28 in the Explain section of this lesson.

193. Find the surface area of the cylinder.

194. Find the surface area of the cylinder.

195. Find the surface area of the cylinder.

196. Find the surface area of the cylinder.

197. Find the surface area of the cylinder. Use 3.14 to
approximate \( \pi \). Round your answer to two decimal places.

198. Find the surface area of the cylinder. Use 3.14 to
approximate \( \pi \). Round your answer to two decimal places.

199. Find the surface area of the cylinder. Use 3.14 to
approximate \( \pi \). Round your answer to two decimal places.

200. Find the surface area of the cylinder. Use 3.14 to
approximate \( \pi \). Round your answer to two decimal places.
201. Find the surface area of the cylinder. Use \( \frac{22}{7} \) to approximate \( \pi \).

\[
\begin{array}{c}
\text{radius} = \frac{1}{4} \text{ in} \\
\text{height} = 4 \frac{1}{4} \text{ in}
\end{array}
\]

202. Find the surface area of the cylinder. Use \( \frac{22}{7} \) to approximate \( \pi \).

\[
\begin{array}{c}
\text{radius} = 1 \frac{3}{4} \text{ in} \\
\text{height} = 7 \frac{1}{2} \text{ in}
\end{array}
\]

203. Find the surface area of the cylinder. Use \( \frac{22}{7} \) to approximate \( \pi \).

\[
\begin{array}{c}
\text{radius} = 5 \frac{3}{4} \text{ cm} \\
\text{height} = 17 \frac{1}{4} \text{ cm}
\end{array}
\]

204. Find the surface area of the cylinder. Use \( \frac{22}{7} \) to approximate \( \pi \).

\[
\begin{array}{c}
\text{radius} = 3 \frac{1}{3} \text{ cm} \\
\text{height} = 17 \frac{1}{3} \text{ cm}
\end{array}
\]

205. Find the surface area of the cylinder.

\[
\begin{array}{c}
\text{diameter} = 4 \text{ ft} \\
\text{height} = 17 \text{ ft}
\end{array}
\]

206. Find the surface area of the cylinder.

\[
\begin{array}{c}
\text{diameter} = 10 \text{ ft} \\
\text{height} = 27 \text{ ft}
\end{array}
\]

207. Find the surface area of the cylinder. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\begin{array}{c}
\text{diameter} = 10.8 \text{ m} \\
\text{height} = 32.3 \text{ m}
\end{array}
\]

208. Find the surface area of the cylinder.

\[
\begin{array}{c}
\text{diameter} = 1.3 \text{ m} \\
\text{height} = 4.6 \text{ m}
\end{array}
\]

209. A farmer decides one day that he needs to repaint his silo. The silo, which is the shape of a cylinder, is 35 feet tall. The area of the silo’s base is 706.5 square feet. How many square feet must the farmer paint in order to give the outside of his silo one new coat of paint, including the top? (Remember that he does not paint the floor!) Use 3.14 to approximate \( \pi \).

210. A farmer decides one day that he needs to repaint his silo. The silo, which is the shape of a cylinder, is 32 feet tall. The area of the silo’s base is 314 square feet. How many square feet must the farmer paint in order to give the outside of his silo one new coat of paint, including the top? (Remember that he does not paint the floor!) Use 3.14 to approximate \( \pi \).

211. A certain soft drink company is about to change its traditional can shape by producing a taller can. The taller can would have a height of 6 inches and a diameter of 2 inches. If the surface area of their current can is 50 square inches, would the taller can have a larger or smaller surface area than the traditional can? By how much? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

212. A certain soft drink company is about to change its traditional can shape by producing a taller can. The taller can would have a height of 5.8 inches and a diameter of 1.9 inches. If the surface area of their current can is 50 square inches, would the taller can have a larger or smaller surface area than the traditional can? By how much? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.
213. An ice cream carton in the shape of a cylinder has a radius of 3.4 inches. The carton is 9.3 inches tall. How many square inches of cardboard are required to produce 3 ice cream cartons complete with lids? Assume that the lid does not overlap the carton, but only rests on top. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

214. An ice cream carton in the shape of a cylinder has a radius of 3.2 inches. The carton is 9.1 inches tall. How many square inches of cardboard are required to produce 4 ice cream cartons complete with lids? Assume that the lid does not overlap the carton, but only rests on top.

215. A cylindrical can of potato chips has a surface area of 91.99 square inches, excluding the lid. If the diameter of the can is 2.4 inches, how tall is the can? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

216. A cylindrical can of potato chips has a surface area of 90.26 square inches, excluding the lid. If the diameter of the can is 2.6 inches, how tall is the can? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

**Volume of a Cylinder**

For help working these types of problems, go back to Examples 29–30 in the Explain section of this lesson.

217. Find the volume of the cylinder.

218. Find the volume of the cylinder.

219. Find the volume of the cylinder.

220. Find the volume of the cylinder.

221. Find the volume of the cylinder. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

222. Find the volume of the cylinder. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

223. Find the volume of the cylinder. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

224. Find the volume of the cylinder. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.
225. Find the volume of the cylinder. Use $\frac{22}{7}$ to approximate $\pi$.

226. Find the volume of the cylinder. Use $\frac{22}{7}$ to approximate $\pi$.

227. Find the volume of the cylinder. Use $\frac{22}{7}$ to approximate $\pi$.

228. Find the volume of the cylinder. Use $\frac{22}{7}$ to approximate $\pi$.

229. Find the volume of the cylinder.

230. Find the volume of the cylinder.

231. Find the volume of the cylinder. Use $3.14$ to approximate $\pi$. Round your answer to two decimal places.

232. Find the volume of the cylinder. Use $3.14$ to approximate $\pi$. Round your answer to two decimal places.

233. The Wilder family decide to drill a well on their property. They find water at a depth of 130 feet. They decide to make the well 4 feet in diameter, leaving them a cylindrical hole. How many cubic feet of dirt had to be removed in order to dig the well? Use $3.14$ to approximate $\pi$. Round your answer to two decimal places.

234. The Holman family decide to drill a well on their property. They hit water at a depth of 240 feet. They decide to make the well 5 feet in diameter, leaving them with a cylindrical hole. How many cubic feet of dirt had to be removed in order to dig the well? Use $3.14$ to approximate $\pi$. Round your answer to two decimal places.

235. A farmer wishes to build a cylindrical silo that will hold all of his grain. County building restrictions allow silos to be no taller than 40 feet. The farmer wants his silo to hold exactly 18000 cubic feet of grain. If the silo is the maximum allowable height, 40 feet, what should the diameter of the silo be? Use $3.14$ to approximate $\pi$. Round your answer to two decimal places.

236. A farmer wishes to build a cylindrical silo that will hold all of his grain. County building restrictions allow silos to be no taller than 36 feet. The farmer wants his silo to hold exactly 16000 cubic feet of grain. If the silo is the maximum allowable height, 36 feet, what should the diameter of the silo be?
237. The height of a cylindrical oatmeal canister is 4 times its radius. If the canister is 12 inches tall, what volume of oats can the container hold? Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

238. The height of a cylindrical potato chip canister is 5.5 times its radius. If the canister has a radius of 1.5 inches, what is the volume of the container? Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

239. A cylindrical oil drum is leaking, forming a puddle of oil at the base of the drum. The leaked oil is collected and found to have a volume of 12000 cubic centimeters. The area of the base of the drum is 1964 square centimeters. After the leak is plugged, the height of the oil in the drum is measured and found to be 60 centimeters. What was the height of the oil before the leak?

240. A cylindrical oil drum is leaking, forming a puddle of oil at the base of the drum. The leaked oil is collected and found to have a volume of 17000 cubic centimeters. The area of the base of the drum is 1812 square centimeters. After the leak is plugged, the height of the oil in the drum is measured and found to be 64 centimeters. What was the height of the oil before the leak?

**Cones**

For help working these types of problems, go back to Examples 31–32 in the Explain section of this lesson.

241. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

242. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

243. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

244. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

245. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

246. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

247. Find the volume of the cone. Use 3.14 to approximate $\pi$.
Round your answer to two decimal places.

Cones

For help working these types of problems, go back to Examples 31–32 in the Explain section of this lesson.
248. Find the volume of the cone. Use \( \frac{22}{7} \) to approximate \( \pi \).

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

249. This cone has a volume of 20.94 cubic centimeters. Find \( x \). Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

250. This cone has a volume of 65.97 cubic cm. Find \( x \). Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

251. This cone has a volume of 821 cubic cm. Find \( x \). Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

252. This cone in has a volume of 70.69 cubic cm. Find \( x \). Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

253. The diameter of the base of a cone is 4 inches. The height of the cone is 9 inches. What is the volume of the cone? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

254. The diameter of the base of a cone is 6 centimeters. The height of the cone is 14 centimeters. What is the volume of the cone? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

255. The diameter of a the base of a cone is 14.2 feet. The height of the cone is 5.6 feet. What is the volume of the cone? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

256. The diameter of the base of a cone is 4 \( \frac{1}{2} \) meters. The height of the cone is 7 \( \frac{1}{4} \) meters. What is the volume of the cone? Use \( \frac{22}{7} \) to approximate \( \pi \).

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

257. A cone-shaped pit is dug in order to pour the foundation of a building and the removed dirt is hauled away. The pit is 15 feet deep and 16 feet across. Unfortunately, the company that was planning to construct the building goes bankrupt and the building project must be canceled. The pit must be refilled with dirt. How many cubic feet of dirt should the workers haul back in order to completely fill in the pit? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]

258. A cone-shaped pit is dug in order to pour the foundation of a building and the removed dirt is hauled away. The pit is 13 feet deep and 14 feet across. Unfortunately, the company that was planning to construct the building goes bankrupt and the building project must be canceled. The pit must be refilled with dirt. How many cubic feet of dirt should the workers haul back in order to completely fill in the pit? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

\[
\text{Volume} = \frac{1}{3} \pi r^2 h
\]
259. An ice shop sells its shaved ice creations in a cone-shaped paper cup. When filling an order, an employee fills the cone with ice all the way to the top, but no higher. If each cone is 5 inches tall and has a radius of 1.5 inches, what volume of ice is needed to fill 5 cones? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

260. An ice shop sells its shaved ice creations in a cone-shaped paper cup. When filling an order, an employee fills the cone with ice all the way to the top, but no higher. If each cone is 4.8 inches tall and has a radius of 1.6 inches, what volume of ice is needed to fill 5 cones?

261. An amusement park has been experiencing a decline in visitors. To draw more customers, the owners of the park decide to order a giant inflatable cone that will fly in the air above the park. If the volume of this giant cone is 14,000 cubic feet and its diameter is 32 feet, how tall is the cone? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

262. An amusement park has been experiencing a decline in visitors. To draw more customers, the owners of the park decide to order a giant inflatable cone that will fly in the air above the park. If the volume of this giant cone is 12,000 cubic feet and its diameter is 30 feet, how tall is the cone? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

263. An ice cream cone is 4.5 inches tall and 2 inches in diameter. A cylindrical container of ice cream is 8 inches tall and the area of its base is 29 square inches. How many ice cream cones can be completely filled using the ice cream from in the cylindrical container? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

264. An ice cream cone is 5 inches tall and 2 inches in diameter. A cylindrical container of ice cream is 9 inches tall and the area of its base is 30 square inches. How many ice cream cones can be completely filled using the ice cream from in the cylindrical container? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

**Spheres**

For help working these types of problems, go back to Examples 33–36 in the Explain section of this lesson.

265. A sphere has a radius of 3 inches. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

266. A sphere has a radius of 5 centimeters. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

267. A sphere has a radius of 8 feet. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

268. A sphere has a radius of 11 meters. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

269. A sphere has a radius of 3.7 inches. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

270. A sphere has a radius of 8.6 feet. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

271. A sphere has a diameter of 6.8 centimeters. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

272. A sphere has a diameter of 10.6 inches. Find its volume. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

273. The volume of a sphere is \( \frac{500\pi}{3} \) cubic centimeters. Find its radius.

274. The volume of a sphere is \( \frac{256\pi}{3} \) cubic inches. Find its radius.

275. The volume of a sphere is 288\( \pi \) cubic feet. Find its radius.

276. The volume of a sphere is \( \frac{5324\pi}{3} \) cubic centimeters. Find its radius.
277. As shown, a rectangular prism has a hemisphere on top. Find the volume of the object. Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

![Rectangular prism with hemisphere](image1)

278. As shown, cylinder has a cone at each end. Find the volume of the object. Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

![Cylinder with cones](image2)

279. A cylindrical hole of diameter 3.8 ft is bored into a rectangular box. Find the volume of the resulting object. Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

![Cylindrical hole in rectangular box](image3)

280. A rectangular prism has two cylinders on top. Find the volume of the resulting object. Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

![Rectangular prism with cylinders](image4)

281. To advertise its grand opening, a sporting goods store plans on placing a giant beach ball in its parking lot. If the beach ball has a radius of 16 feet and Joe exhales at a rate of 15 cubic feet per minute, how long will it take Joe to blow up the beach ball? Use 3.14 to approximate $\pi$. Round your answer to two decimal places.

282. To advertise its grand opening, a sporting goods store plans on placing a giant beach ball in its parking lot. If the beach ball has a radius of 15 feet and Joe exhales at a rate of 17 cubic feet per minute, how long will it take Joe to blow up the beach ball?

283. The radius of a tennis ball is 4 centimeters. The radius of a golf ball is 2.5 centimeters. In terms of volume, how many times larger than the golf ball is the tennis ball? Use 3.14 to approximate $\pi$. Round your answer to two decimal places.
284. The radius of a basketball is 4.5 inches. The diameter of a volleyball is 7 inches. In terms of volume, how many times larger than the volleyball is the basketball? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

285. A weightlifter trains using a steel weight that consists of a cylindrical rod with 2 spheres at each end. The rod is 37 inches long and has a radius of 0.8 inches. Each sphere has a radius of 4.5 inches. What is the volume of the steel weight? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

286. A weightlifter trains using a steel weight that consists of a cylindrical rod with 2 spheres at each end. The rod is 43 inches long and has a radius of 0.9 inches. Each sphere has a radius of 5.2 inches. What is the volume of the steel weight? Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

287. The Kingdome consists of a cylinder with half of a sphere placed on top. The height of the Kingdome, measured from the center of the floor to the top of the dome, is 186 feet. The height of the cylindrical portion of the Kingdome is 120 feet. The radius of the cylinder is the same as the radius of the domed portion. Use this information to calculate the volume of the Kingdome. A labeled sketch is quite helpful. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.

288. The Astrodome consists of a cylinder with half of a sphere placed on top. The height of the Astrodome, measured from the center of the floor to the top of the dome, is 190 feet. The height of the cylindrical portion of the Astrodome is 115 feet. The radius of the cylinder is the same as the radius of the domed portion. Use this information to calculate the volume of the Astrodome. A labeled sketch is quite helpful. Use 3.14 to approximate \( \pi \). Round your answer to two decimal places.
Take this Practice Test to prepare for the final quiz in the Evaluate module of this lesson on the computer.

**Practice Test**

1. Find the perimeter and the area of the parallelogram shown.

   ![Parallelogram Diagram]

2. a. Find the area of the triangle shown.

   ![Triangle Diagram]

   b. Find the area of the rectangle shown.

   ![Rectangle Diagram]

3. A given trapezoid has two parallel sides, its bases, with lengths 35.1 in. and 23.7 in. The height of the trapezoid is 12 in. Find the area of the trapezoid.

4. The diameter of a circle is 14 centimeters. Find the circumference and the area of this circle. Use 3.14 to approximate $\pi$. Round your answers to two decimal places.

5. Find the volume and the surface area of the rectangular prism shown.

   ![Rectangular Prism Diagram]
6. The height of the cylinder shown is 29 cm. The diameter of each circular base is 18 cm. Which of the following is the surface area of the cylinder?

![Cylinder Diagram]

a. $684\pi$ cm$^2$  
b. $9396\pi$ cm$^3$  
c. $162\pi + 522$ cm$^3$

7. The height of the cone shown is 15 cm. The diameter of its circular base is 12 cm. Find the volume, $V$, of the cone. Use 3.14 to approximate $\pi$.

![Cone Diagram]

8. A sphere has a radius of 7 inches. Choose the value below that best approximates the volume of the sphere. Use 3.14 to approximate $\pi$.

a. 2872.05 cm$^3$  
b. 457.33 cm$^3$  
c. 1436.03 cm$^3$  
d. 1372 cm$^3$